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NOTE

Single-Firm Event Studies, Securities Fraud, and Financial Crisis: Problems of Inference

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Abstract. Lawsuits brought pursuant to section 10(b) of the Securities and Exchange Act depend on the reliability of a statistical tool called an event study to adjudicate issues of reliance, materiality, loss causation, and damages. Although judicial acceptance of the event study technique is pervasive, there has been little empirical analysis of the ability of event studies to produce reliable results when applied to a single company's security.

Using data from the recent financial crisis, this Note demonstrates that the standard-model event study used in most court proceedings can lead to biased inferences sanctioned through the Daubert standard of admissibility for expert testimony. In particular, in the presence of broad market volatility, a base event study will cause too many returns to be identified as statistically significant. Even recently proposed variations of the event study model specifically designed to address violations of the statistical assumptions of an event study will not completely correct this bias. This Note proposes two alternative forms of event studies that are capable of creating statistically reliable results and should be adopted by courts in instances where there is cause to believe that market volatility has increased.

Over previous decades, the judiciary has steadily moved toward a reliance on empirics and expert testimony in overseeing complex civil cases. Yet there has been surprisingly little research accompanying this judicial deference on the ability of statistical evidence to produce the promised result. This Note calls into question whether this movement has been beneficial from a logical or empirical perspective, but it demonstrates that alternative techniques that can aid the finder of fact in resolving these disputes—regardless of market trends—may in fact exist.

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Introduction

An event study is a technique used to analyze the effect of a predetermined "event" on the value of a company's security.[1] The event effect is determined by comparing the actual return of the security to that predicted by an econometric model incorporating changes in a market index and the security's historical comovement with the market. Given the technique's ability to isolate firm-specific movements in the price of a company's security, modern courts effectively require a plaintiff to provide a methodologically sound event study to prevail on both a class certification motion and the merits.

Event studies are appropriated from a larger literature in financial economics, in which they are traditionally used over a broad set of securities for a specific form of event that generally occurs across time periods.[2] The statistical assumptions underlying interpretation in this context are often robust to the typical econometric concern of model choice. While judicial reliance on the event study has progressed inexorably, surprisingly little research has been devoted to analyzing the statistical properties and suitability of an event study used for a single security and for a limited number of events.

Early articles comparing different event study techniques found model performance to be indifferent to methodological choice.[3] However, financial economists have long been aware that increases in market volatility can lead to biased tests of statistical significance and corresponding difficulties in interpretation.[4] Beginning in August 2007, an unprecedented credit crisis hit U.S. financial markets,[5] causing a significant spike in overall market volatility. Based on an empirical analysis of the results of competing event study models over the crisis period, this Note demonstrates that standard methods for analyzing the returns of a single security generate too many statistically significant excess returns, which will cause courts to find event effects where none may exist.[6] However, there are alternative models capable of providing results robust to increased security variance by explicitly controlling for changes in marketwide volatility. This suggests that courts should approach

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1. *See* Michael J. Kaufman & John M. Wunderlich, *Regressing: The Troubling Dispositive Role of Event Studies in Securities Fraud Litigation*, 15 STAN. J.L. BUS. & FIN. 183, 188 (2009).
2. *See* S.P. Kothari & Jerold B. Warner, *Econometrics of Event Studies*, *in* 1 HANDBOOK OF CORPORATE FINANCE: EMPIRICAL CORPORATE FINANCE 3, 8-9 (B. Espen Eckbo ed., 2007).
3. *See* Stephen J. Brown & Jerold B. Warner, *Measuring Security Price Performance*, 8 J. FIN. ECON. 205, 249 (1980).
4. *See, e.g.*, Stephen J. Brown & Jerold B. Warner, *Using Daily Stock Returns: The Case of Event Studies*, 14 J. FIN. ECON. 3, 23 (1985).
5. *See* Jill Treanor, *Credit Crunch Pinpointed to 9 August 2007—The Day the World Changed*, GUARDIAN (Dec. 1, 2011, 3:49 PM EST), http://gu.com/p/33npx/stw.
6. See Part II.A below for an overview of event studies and their use in securities litigation.

unadjusted event study results with caution when provided by expert witnesses to explain security performance over periods with known changes in market volatility.

The consequence of accepting biased event study results is magnified by the increased reliance on empirics in adjudicating complex legal disputes. Over the past several decades, courts have relinquished many tasks traditionally confined to the judiciary in favor of ostensibly objective statistical analysis.[7] In order to understand the interplay between event study analysis and securities fraud doctrine, the structure of this Note is as follows: Part I describes the historical development of the modern securities fraud class action, Part II explains the role of expert testimony in the disposition of a suit, Part III details the extant literature on event studies, and Part IV provides a description of the data and empirical methodology used in this Note to compare event study models. In particular I will use both Type I and Type II error tests to compare the specification of competing event study models. Part V presents the results of the comparative event study model analysis.

I. Historical Development of Securities Fraud Lawsuits

A general understanding of the history and theory underlying modern doctrine is necessary to appreciate the prominent role performed by event studies in the securities fraud framework. The structure of securities class action suits developed through decades of statutory enactment, judicial experimentation, and evolving economic theory. The resulting standard for a cause of action depends critically on an empirical analysis of asset price guided by the tenets of a debatable economic theory. Due to this reliance on econometric analysis, courts require expert economic testimony to satisfy the majority of the factual determinations of a case—with a particular reliance on

7. See Parts I.B and I.A.2 below for a discussion of how reliance and materiality in securities litigation are now essentially empirical questions. *See also* Thomas J. Campbell, *Regression Analysis in Title VII Cases: Minimum Standards, Comparable Worth, and Other Issues Where Law and Statistics Meet*, 36 STAN. L. REV. 1299, 1311 (1984) (noting that the Supreme Court sanctioned a "two-standard-deviations rule" for assessing the representation of minority groups in employment discrimination cases); D.H. Kaye, *Statistical Analysis in Jury Discrimination Cases*, 25 JURIMETRICS J. 274 (1985) (describing the use of statistical evidence to test the constitutionality of jury composition under the Fifth, Sixth, Seventh, and Fourteenth Amendments). Recently there has been judicial pushback, as judges have questioned the probativeness of statistical evidence in certain contexts. *See, e.g.*, Wal-Mart Stores, Inc. v. Dukes, 131 S. Ct. 2541, 2555 (2011) (discounting statistical evidence of companywide employment discrimination that incorporated evidence of disparate impact and aggregated regional and national data); Exxon Shipping Co. v. Baker, 554 U.S. 471, 501 n.17 (2008) (refusing to rely on experimental evidence using "mock juries" that was partially funded by the defendant). I thank Dan Ho for bringing this to my attention.

the use of event studies in establishing market efficiency, price distortion, and loss causation.

A. Statutory Underpinnings and the Judicial Creation of a Private Cause of Action

Private securities litigation is grounded in regulations enacted to ensure open and transparent securities markets.[8] Congress implemented two critical articles of legislation in the aftermath of the stock market crash of 1929: the Securities Act of 1933[9] and the Securities Exchange Act of 1934.[10] The Securities Act applies standards for the registration and distribution of securities, while the Exchange Act regulates secondary trading markets[11] and includes the continuous, periodic reporting requirements for securities issued under various Securities and Exchange Commission (SEC) provisions.[12] The overarching objective of both statutes is to guarantee the "full and fair disclosure" of information critical to the integrity of the market.[13]

Although suits do arise under the Securities Act, particularly section 11,[14] section 10 of the Exchange Act has become the statutory workhorse for private suits alleging fraudulent misstatements or omissions.[15] Section 10(b) of the Exchange Act stipulates that it is unlawful

> [t]o use or employ, in connection with the purchase or sale of any security registered on a national securities exchange or any security not so registered, or any securities-based swap agreement any manipulative or deceptive device or contrivance in contravention of such rules and regulations as the [Securities and

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8. *See* John Hanna, *The Securities Exchange Act as Supplementary of the Securities Act*, 4 LAW & CONTEMP. PROBS. 256, 256-57 (1937).
9. Pub. L. No. 73-22, tit. I, 48 Stat. 74 (codified as amended at 15 U.S.C. §§ 77a-77aa (2014)).
10. Pub. L. No. 73-291, 48 Stat. 881 (codified as amended at 15 U.S.C. §§ 78a-78pp). Although both the Securities Act and the Securities Exchange Act (or "Exchange Act") have been modified over the years, their role in the regulatory framework of the securities market has remained constant.
11. *See* Hanna, *supra* note 8, at 256-57.
12. *See* Stefan J. Padfield, *Who Should Do the Math?: Materiality Issues in Disclosures that Require Investors to Calculate the Bottom Line*, 34 PEPP. L. REV. 927, 931 (2007).
13. *Id.*
14. *See* Frederick C. Dunbar & Dana Heller, *Fraud on the Market Meets Behavioral Finance*, 31 DEL. J. CORP. L. 455, 459 (2006) (noting that section 11 is the most commonly used provision of the Securities Act and ascribes liability for misleading statements or omissions of material facts in the registration statement of a security); *see also* Securities Act of 1933 § 11, 15 U.S.C. § 77k.
15. *See* Jennifer J. Johnson, *Secondary Liability for Securities Fraud: Gatekeepers in State Court*, 36 DEL. J. CORP. L. 463, 465 (2011) (noting that section 10(b) of the Exchange Act and Rule 10b-5, which was promulgated under section 10(b), are "the most widely utilized antifraud provisions in the federal securities laws").

> Exchange] Commission may prescribe as necessary or appropriate in the public interest or for the protection of investors.[16]

The benefits of this section of the statute are subtle but significant—Congress intended for it to function as a "catch-all" provision allowing the SEC to expand its authority into evolving realms of fraudulent practice.[17] Although there was no congressional intent to provide a private cause of action in passing the Exchange Act, federal courts interpreted such a right as "implied in the words of the statute and its implementing regulation."[18]

In 1942, consistent with the requirements of section 10(b), the SEC promulgated Rule 10b-5, which made it unlawful to "make any untrue statement of a material fact or to omit to state a material fact necessary in order to make the statements made, in the light of the circumstances under which they were made, not misleading."[19] In addition to being false and material, the action or omission giving rise to a Rule 10b-5 violation must also be made with the statutorily required state of mind, and the false statement at issue must generate detrimental reliance—the kind of reliance that leads to tangible loss.[20] The objective in passing Rule 10b-5 was to extend the SEC's regulatory power to postoffering transactions, although there is again no evidence of a desire to expand the scope of the rule to private civil remedies.[21]

Despite a lack of explicit congressional or administrative intent, federal courts began inferring a private right of action under Rule 10b-5 in 1946.[22] Later, in the influential Second Circuit decision *SEC v. Texas Gulf Sulphur Co.*, the court categorically abandoned a privity requirement, allowing private actors to sue a corporation for damages suffered as a result of third-party

16. Securities Exchange Act of 1934 § 10(b), 15 U.S.C. § 78(j).

17. Joseph A. Grundfest, *Damages and Reliance Under Section 10(b) of the Exchange Act*, 69 BUS. LAW. 307, 321 (2014).

18. *Id.* at 321 & n.66 (quoting Stoneridge Inv. Partners, LLC v. Scientific-Atlanta, Inc., 552 U.S. 148, 157 (2008)).

19. 17 C.F.R. § 240.10b-5 (2015).

20. *See* Padfield, *supra* note 12, at 932.

21. Grundfest, *supra* note 17, at 312. Although this appears to be true at the time of passage, some have noted that the landmark opinion codifying the fraud-on-the-market doctrine, which removed the need to prove individual reliance in class actions, under Rule 10b-5, *Basic Inc. v. Levinson*, 485 U.S. 224 (1988), was itself largely authored by the SEC in conjunction with the Solicitor General's office. The key analytical points regarding materiality and reliance appear to have been taken directly from an amicus brief filed on behalf of the SEC. Donald C. Langevoort, Basic *at Twenty: Rethinking Fraud on the Market*, 2009 WIS. L. REV. 151, 157.

22. Grundfest, *supra* note 17, at 322. As discussed by the Supreme Court in *Herman & MacLean v. Huddleston*, 459 U.S. 375, 380 n.10 (1983), the first case incorporating the implied right of action was *Kardon v. National Gypsum Co.*, 69 F. Supp. 512, 513-14 (E.D. Pa. 1946). *See also* Grundfest, *supra* note 17, at 322 n.75.

transactions.[23] Over subsequent decades, judicial acceptance of an implied private right of action spread across jurisdictions and was ultimately upheld by the Supreme Court in *Herman & MacLean v. Huddleston*.[24] Justice Marshall, writing for the majority, acquiesced to federal judicial practice, while noting that the Securities Act and the Exchange Act had overtly created other private actions while failing to do so here. Having been consistently recognized for over thirty-five years, the existence of an implied right of action was now "simply beyond peradventure."[25] The Court was similarly persuaded that given the opportunity to clarify congressional intent while enacting significant revisions to the nation's securities laws in 1975, Congress tacitly registered its approval of a private right of action under section 10(b).[26]

Initially, the burden of proof for actions brought under section 10(b) mirrored that of common law deceit.[27] To recover money damages, plaintiffs had to demonstrate materiality (that the misstatement or omission was in fact relevant to a rational investor), scienter (an intent to deceive on behalf of the organization or its agent), reliance (inducement to trade as a result of the misstatement or omission), and loss causation (that the misstatement or omission constituted the proximate cause of the complaining party's injuries).[28] However, structural limitations in adapting the common law standard made the consolidation of securities fraud claims unworkable. Each individual plaintiff's damage amount was unlikely to be large enough to justify the litigation expenses associated with a civil trial. It would also be unduly burdensome to require each plaintiff to prove direct reliance on the purportedly fraudulent misstatement. To create a practical standard for consolidating suits, courts looked to the burgeoning academic consensus of "market efficiency" within the field of financial economics regarding the rationality of financial markets.

B. Market Efficiency, "Fraud-on-the-Market," and *Basic Inc. v. Levinson*

Although many commentators consider the Supreme Court's decision in *Basic Inc. v. Levinson*[29] to have ushered in the prevailing standard for securities fraud class actions, *Basic* did not actually represent a marked departure from

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23. 401 F.2d 833, 861 (2d Cir. 1968) (en banc).
24. 459 U.S. 375.
25. *Id.* at 380.
26. *Id.* at 384-86.
27. *See* Dunbar & Heller, *supra* note 14, at 458.
28. *Id.*
29. 485 U.S. 224 (1988).

prior judicial practice.[30] From the outset, the judiciary recognized that it would be functionally impossible to require plaintiffs to demonstrate individual reliance.[31] While courts had expressed a general preference for a presumption of individual reliance, a coherent framework for minimizing the reliance burden on plaintiffs would allow class action treatment of Rule 10b-5 violations to continue as established practice.[32] Perhaps surprisingly, such a theory was found within the conservative law and economics movement, which advocated for applying the principles of the theory of capital market efficiency—that all available public information reflecting the firm's prospects are reflected in the prevailing price of the security—to the standards of materiality and reliance embedded in Rule 10b-5.[33]

The efficient market hypothesis began as a scholarly attempt to answer a question that had long bedeviled individual investors: Is it possible to systematically beat the market? Beginning with seminal articles written by future Nobel Laureates Paul Samuelson and Eugene Fama in the 1960s, many economists were persuaded that asset prices on liquid markets fluctuated randomly.[34] In practical terms, this meant that the daily price changes of large common stocks were unpredictable, rendering it impossible for investors to achieve above-average returns without a willingness to take on higher risk.[35] Although some scholars registered skepticism with the hypothesis, the general

30. *See* Jill E. Fisch, *The Trouble with* Basic*: Price Distortion After* Halliburton, 90 WASH. U. L. REV. 895, 900 (2013).

31. *Id.* at 900 & n.27.

32. *See* Rosemary J. Thomas, Note, *The Fraud-on-the-Market Theory: A "Basic"ally Good Idea Whose Time Has Arrived,* Basic, Inc. v. Levinson, 22 IND. L. REV. 1061, 1063 (1989) (claiming that the fraud-on-the-market presumption endorsed in *Basic* "serves as an entree for plaintiff class actions" by removing a burdensome procedural hurdle).

33. *See* Langevoort, *supra* note 21, at 154.

34. Eugene F. Fama, *The Behavior of Stock-Market Prices,* 38 J. BUS. 34 (1965); Paul A. Samuelson, *Proof that Properly Anticipated Prices Fluctuate Randomly,* INDUS. MGMT. REV., Spring 1965, at 41. Although these articles are generally viewed as the intellectual precursors to the efficient capital markets hypothesis, a similar observation was in fact made by the French mathematician Louis Bachelier in 1900. Edward J. Sullivan & Timothy M. Weithers, *Louis Bachelier: The Father of Modern Option Pricing Theory,* 22 J. ECON. EDUC. 165, 166 (1991). For an interesting review of the intellectual history of the doctrine and its connection to the debate surrounding the theory's role in the recent financial crisis, see JUSTIN FOX, THE MYTH OF THE RATIONAL MARKET: A HISTORY OF RISK, REWARD, AND DELUSION ON WALL STREET (2d ed. 2010).

35. *See* Burton G. Malkiel, *The Efficient Market Hypothesis and Its Critics,* J. ECON. PERSP., Winter 2003, at 59, 59-60 (noting that efficient markets are associated with returns "where all subsequent price changes represent random departures from previous prices" and "do not allow investors to earn above-average returns without accepting above-average risks").

consensus remained that markets approached efficiency.[36] In recent years, the field of behavioral finance—arguing instead that recognized psychological biases often lead to systematic mispricing of financial instruments—has amassed growing appeal but has yet to have a corresponding effect on judicial opinion.[37]

Adherents to the efficient market hypothesis saw an opportunity for theory to ameliorate the inherent legal ambiguities associated with securities fraud cases. If one takes the position that prices reflect all publicly available information, then questions of individual reliance become irrelevant. As Dan Fischel, a leading conservative law and economics scholar and adherent to this view, wrote in an influential 1982 article in the *Business Lawyer*, "Because all publicly available information is embedded in stock prices, investors who accept the market price are fully protected."[38] When stocks are priced to accurately reflect all public information, individual investors have little incentive to seek out private information. Instead of protecting individual investors, "[t]he law should protect markets: markets will then protect investors."[39] In fact, according to Fischel, the reliance requirement should have been removed from securities fraud doctrine in its entirety:

> Because the rational course for investors is simply to accept (rely on) the market price, it is of no consequence whether a plaintiff can demonstrate that he relied upon a particular piece of information. If fraudulent conduct caused the market price to be artificially high or low, a plaintiff under the market model has been injured even if he was totally unaware of the challenged conduct.[40]

Rather than totally abandoning the reliance requirement, courts have embraced insights from the law and economics movement to develop an alternative scheme through which plaintiffs can avoid subjective determinations of individual reliance.[41] This "fraud-on-the-market" (FOTM) theory posits that defendants distort their stock price through the use of deceptive misstatements or omissions, effectively inducing the plaintiff's

36. *See* Daniel R. Fischel, *Use of Modern Finance Theory in Securities Fraud Cases Involving Actively Traded Securities*, 38 BUS. LAW. 1, 4 n.9 (1982).

37. In *Amgen Inc. v. Connecticut Retirement Plans & Trust Funds*, a cryptic concurrence by Justice Alito signaled that the Court may be willing to rethink its adherence to a "faulty economic premise." 133 S. Ct. 1184, 1204 (2013) (Alito, J., concurring). However, in the recent installment of *Halliburton Co. v. Erica P. John Fund, Inc.* (*Halliburton II*), the Court decided against overturning the *Basic* presumption on market efficiency grounds, leaving the doctrine's acceptance of its principles in place. 134 S. Ct. 2398, 2406-10 (2014).

38. Fischel, *supra* note 36, at 5.

39. Langevoort, *supra* note 21, at 165.

40. Fischel, *supra* note 36, at 8.

41. *See* Fisch, *supra* note 30, at 907.

reliance.[42] The FOTM theory was first adopted by the District Court for the Southern District of New York in 1969[43] and by the Ninth Circuit Court of Appeals in 1975.[44] By the time *Basic* came before the Supreme Court, "all courts of appeals that had considered the question had invoked some kind of reliance presumption in order to make fraud-on-the-market class-action lawsuits certifiable."[45]

While *Basic* also set the prevailing judicial standard for materiality, the "more profound and more enigmatic" determination made by the Court involved reliance and the FOTM theory.[46] In a 4-2 ruling, the majority rejected the argument that the FOTM doctrine eliminated the reliance requirement, claiming instead that "[r]eliance provides the requisite causal connection between a defendant's misrepresentation and a plaintiff's injury."[47] Citing recent empirical studies, the Court declared that the price of a security traded on a well-developed market "reflects all publicly available information" and found a presumption of reliance to be "supported by common sense and probability."[48] An investor purchasing shares "does so in reliance on the integrity of that [market] price," and as such, reliance on the purported material misrepresentations could be presumed under Rule 10b-5.[49]

C. Post-*Basic* Case Law and the Structure of the FOTM Class Action

Following the ruling in *Basic*, the modern structure of a Rule 10b-5 securities class action took shape. To establish the presumption of reliance, plaintiffs would ultimately be required to demonstrate both that the affected security traded in an "efficient market"[50] and that the misrepresented or

42. *Id.*
43. *See* Herbst v. Able, 47 F.R.D. 11, 16 (S.D.N.Y 1969).
44. *See* Blackie v. Barrack, 524 F.2d 891, 906 (9th Cir. 1975).
45. Langevoort, *supra* note 21, at 153.
46. *Id.*
47. Basic Inc. v. Levinson, 485 U.S. 224, 243 (1988).
48. *Id.* at 246.
49. *Id.* at 247.
50. Although this could seemingly be read to indicate that the *exchange* on which the security traded needed to be characterized as open or developed, *see id.* ("[N]early every court that has considered the proposition has concluded that where materially misleading statements have been disseminated into an impersonal, *well-developed market for securities*, the reliance of individual plaintiffs . . . may be presumed." (emphasis added)), courts instead adopted a standard affirming that the market for the *individual security* from which the suit was derived needs to be characterized as efficient, *see, e.g.*, Cammer v. Bloom, 711 F. Supp. 1264, 1277 (D.N.J. 1989).

omitted information was material.[51] Additionally, there must be an established causal link between the purported misrepresentations and the plaintiff's ultimate loss[52] and a proper calculation of classwide damages upon positive disposition on the merits.[53] Each of these factual determinations would become empirical prerequisites, requiring the provision of expert testimony and econometric analysis.

1. Market efficiency

Although *Basic* states that market efficiency is a precondition for the reliance presumption, the Court did not specify how "efficient" the market must be or the manner in which market efficiency could be tested.[54] The concept of efficiency, intrinsically connected to the theory underlying the FOTM doctrine, had been discussed in precedent but rarely applied rigorously.[55] Although this inquiry would appear ripe for Supreme Court guidance, it was ultimately left to lower courts to establish a workable standard for determining whether the market for a security was efficient enough to establish the reliance presumption.

The most influential standard adopted in testing market efficiency was articulated in *Cammer v. Bloom*,[56] decided in New Jersey's district court shortly after *Basic*. The court created a list of conditions (today colloquially known as the "*Cammer* factors"[57]) to guide the determination of whether the market for a particular stock is legally efficient.[58] These factors include the weekly trading volume,[59] the presence of "a significant number of securities analysts"

51. *See* Grundfest, *supra* note 17, at 327. Grundfest notes that, although this presumption is nominally rebuttable, it is effectively unrebuttable in practice. At the time of writing, he was able to identify only six instances in which defendants rebutted the presumption of reliance. *Id.* at 388.

52. *See* Fisch, *supra* note 30, at 914.

53. Miller v. Asensio & Co., 364 F.3d 223, 235 (4th Cir. 2004) (holding that compensable damages must be independently proven even where liability has been established).

54. *See* Langevoort, *supra* note 21, at 166.

55. *See* Barbara Black, *The Strange Case of Fraud on the Market: A Label in Search of a Theory*, 52 ALB. L. REV. 923, 937 (1988).

56. 711 F. Supp. 1264 (D.N.J. 1989).

57. *See, e.g.*, Bradford Cornell, *Market Efficiency and Securities Litigation: Implications of the Appellate Decision in* Thane, 6 VA. L. & BUS. REV. 237, 245-46 (2011).

58. Although there are variants of market efficiency proposed in the financial economics literature, courts have adopted the semi-strong form of market efficiency, which stipulates that the price of the security incorporates all available public information. *See* Daniel R. Fischel, *Efficient Capital Markets, the Crash, and the Fraud on the Market Theory*, 74 CORNELL L. REV. 907, 911 (1989).

59. *Cammer*, 711 F. Supp. at 1286. Citing to a securities treatise, the court noted that total weekly trading turnover of at least two percent would be indicative of a security

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following and reporting on the company's financial position,[60] the existence and number of market makers and arbitrageurs,[61] an entitlement to file an S-3 registration statement with the SEC,[62] and empirical evidence of a cause-and-effect relationship between unexpected corporate events and movements in the price of the security.[63] Some courts have added additional elements to the test, including the market capitalization of the security, the bid-ask spread, the percentage of stock held by insiders, and the presence of institutional investors trading in the security.[64]

The ability of any of these judicially constructed proxies to reveal the degree of efficiency in a security has been called into question from the outset. Bradford Cornell and James C. Rutten note that only the number of analysts following a stock and the cause-and-effect relationship between news and price "directly speak to whether a market is efficient."[65] The remaining factors are better seen as indicia of efficiency; they may correlate with the notion of market efficiency as understood by financial economists, but in isolation they do not influence the mechanism through which the price of a stock comes to reflect its fundamental value. Courts have consequently treated the fifth *Cammer* factor, a cause-and-effect relationship between unexpected corporate events and the movement in asset price, as the primary test of efficiency.[66] As

trading in an efficient market. *Id.* (citing 4 ALAN R. BROMBERG & LEWIS D. LOWENFELS, BROMBERG AND LOWENFELS ON SECURITIES FRAUD & COMMODITIES FRAUD § 8.6 (2d ed. 1988)).

60. *Id.* The court hypothesized that the presence of securities analysts would create more accurate pricing of the security as the "stock would be bid up or down to reflect the financial information contained in [financial] reports." *Id.*

61. *Id.* The presence of market makers, who "react swiftly to company news and reported financial results," would "ensure completion of the market mechanism." *Id.* at 1286-87.

62. *Id.* at 1287. The court's reasoning here has more to do with the size of the market for the security than any procedural impact the SEC filing may have. Thus, this requirement may be satisfied if the ineligibility to file an S-3 was solely a result of "timing factors." *Id.* The court here presumes that the number and value of shares outstanding "imply efficiency." *Id.*

63. *Id.*

64. William O. Fisher, *Does the Efficient Market Theory Help Us Do Justice in a Time of Madness?*, 54 EMORY L.J. 843, 862 (2005).

65. Bradford Cornell & James C. Rutten, *Market Efficiency, Crashes, and Securities Litigation*, 81 TUL. L. REV. 443, 454 (2006).

66. *In re* Winstar Commc'ns Sec. Litig., 290 F.R.D. 437, 448 (S.D.N.Y. 2013) ("The fifth *Cammer* factor that courts consider is whether the Plaintiff can demonstrate empirical facts that show 'a cause and effect relationship between unexpected corporate events or financial releases and an immediate response in the [security's] price.' Courts have considered this 'the most important *Cammer* factor' because without finding this causal relationship, it is 'difficult to presume that the market will integrate the release of material information about a security into its price.'" (alteration in original) (citations

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the *Cammer* court observed, "This, after all, is the essence of an efficient market and the foundation for the fraud on the market theory."[67]

Some academics and practitioners have called for removing the efficiency requirement from the FOTM doctrine.[68] Others maintain that efficiency is still a necessary predicate for a presumption of reliance,[69] particularly when viewed in conjunction with the theory of damages.[70] Regardless of academic dispute, the judiciary still considers market efficiency to be a certification requirement. In 2005, the First Circuit grappled with this issue in *In re Polymedica Corp. Securities Litigation*.[71] Although acknowledging the unsettled nature of the case law, *Polymedica* held that the market for the company's stock has to be one "in which the market price of the stock *fully reflects all* publicly

omitted) (first quoting *Cammer*, 711 F. Supp. at 1287; then quoting Teamsters Local 445 Freight Div. Pension Fund v. Bombardier Inc., 546 F.3d 196, 207 (2d Cir. 2008))).

67. *Cammer*, 711 F. Supp. at 1287.

68. *See, e.g.*, Langevoort, *supra* note 21, at 176 ("If *Basic*'s presumption is essentially an entitlement to rely on the market price as undistorted by fraud, it is hard to see why investors should lose that entitlement simply because of some market imperfection."); Jonathan R. Macey et al., *Lessons from Financial Economics: Materiality, Reliance, and Extending the Reach of* Basic v. Levinson, 77 VA. L. REV. 1017, 1018 (1991) ("Though restricting fraud-on-the-market theory to efficient markets is intuitively appealing[,] . . . we believe this distinction between efficient and inefficient markets to be specious. We suggest that the focus of the Supreme Court's holding in *Basic* is misplaced: what determines whether investors were justified in relying on the integrity of the market price is not the efficiency of the relevant market but rather whether a misstatement distorted the price of the affected security.").

69. *See, e.g.*, Dunbar & Heller, *supra* note 14, at 532 ("If one accepts that certain actively-traded securities at certain times do not obey the rules of an efficient market and, as a result, investors may not rely on the price to fully reflect publicly available information, then it is difficult to understand why the presumption of reliance should not be rejected just as it is for illiquid securities that do not obey the rules of an efficient market. Failing to reject the presumption of reliance in such a case would be tantamount to changing the fraud-on-the-market theory from a presumption to removing plaintiffs' burden of proving reliance altogether. This would make *Basic* unintelligible because the Court left open the possibility that reliance could be disproved; a position that at this time does not seem to have adverse policy consequences.").

70. *See, e.g.*, Cornell & Rutten, *supra* note 65, at 449 ("[I]n the damages context, we argue for a much stricter standard for efficiency that is again tied to the fundamental issue. . . . Damages cannot accurately be measured by reference to the decline in the stock price unless the market is *perfectly* efficient such that it reacts *perfectly* to fraudulent statements and the later revelation of true facts. . . . Although damages in securities fraud cases, as in other types of cases, need not be measured accurately and only approximated, even approximating damages by reference to the decline in the stock price would require the market to approximate perfect efficiency because even minor inefficiencies are magnified significantly by selection bias." (footnote omitted)).

71. 432 F.3d 1 (1st Cir. 2005).

available information" for the presumption to apply.[72] More recently, many believed that the Supreme Court's conservative leaning would result in a radical restructuring of the class action device.[73] However, in *Halliburton II* the Court refused to jettison the efficiency requirement, but did clarify that the standard should be generalized efficiency.[74] It would appear as though the efficient market requirement will remain while the FOTM theory is in place.

2. Materiality, price distortion, and loss causation

In addition to the presumption of reliance, plaintiffs bear the burden of demonstrating that the alleged misstatements or omissions were material to the average investor. In *Basic*, the Supreme Court unanimously[75] affirmed the position on materiality previously established in *TSC Industries, Inc. v. Northway, Inc.*: in order to fulfill the materiality requirement, "there must be a substantial likelihood that the disclosure of the omitted fact would have been viewed by the reasonable investor as having significantly altered the 'total mix' of information made available."[76] The Court refused to follow the lower court standard that based the materiality determination on policy factors, such as the protection of corporate secrets.[77] Instead, the Court held the materiality inquiry should involve a fact-specific analysis of whether a reasonable investor would hold the particular alleged misrepresentation or omission to be significant in the context of the information available to the market.[78] Much like the requirement for market efficiency, this standard proved long on rhetoric but short on practical application.

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72. *Id.* at 14.

73. *See, e.g.*, Grundfest, *supra* note 17, at 310; *see also* Paul Atkins, *The Supreme Court's Opportunity to End Abusive Class Action Securities Lawsuits*, FORBES (Mar. 4, 2014, 3:57 PM), http://www.forbes.com/sites/realspin/2014/03/04/the-supreme-courts-opportunity-to-end-abusive-securities-class-action-lawsuits (calling for the Supreme Court to "clear the judicial underbrush surrounding securities class action suits" that *Basic* caused).

74. *Halliburton II*, 134 S. Ct. 2398, 2410 (2014). Donald Langevoort believes that this focus on generalized efficiency may remove much of the pressure to analyze market efficiency at the class certification stage. *See* Donald C. Langevoort, *Judgment Day for Fraud-on-the-Market: Reflections on* Amgen *and the Second Coming of* Halliburton, 57 ARIZ. L. REV. 37, 52-53 (2015).

75. Although this was a unanimous decision, the bench was not full. Justice Powell had retired a few months after certiorari was granted, and his successor, Justice Kennedy, had yet to be sworn in. Chief Justice Rehnquist and Justice Scalia had also recused themselves. Langevoort, *supra* note 21, at 157.

76. Basic Inc. v. Levinson, 485 U.S. 224, 231-32 (1988) (quoting TSC Indus., Inc. v. Northway, Inc., 426 U.S. 438, 449 (1976)).

77. Langevoort, *supra* note 21, at 152.

78. *Basic*, 485 U.S. at 240.

Law and economics scholars saw an opportunity for the efficient market hypothesis to categorically determine materiality. Notably skeptical of a factfinder's ability to discern the particular significance of an information set to the average investor, scholars proposed allowing the market to make the determination. According to Fischel, "The primary advantage of the market model is that it recognizes that the question of what information is important to investors cannot be answered in the abstract."[79] Materiality should be determined solely on the basis of whether "the alleged misrepresentation or disclosure caused the security to trade at an artificially high or low price."[80]

As with reliance, courts have been receptive to this interpretation of the legal standard.[81] In *In re Merck & Co. Securities Litigation*, the Third Circuit stated that materiality "may be measured post hoc" by looking at the movement of the stock price in the period immediately after the disclosure of information.[82] This mode of reasoning was in fact "part of a larger agenda" within the conservative law and economics movement to supplant subjective evaluations of materiality with an impartial market-based standard.[83] It was the promise of inherent objectivity and a "rigorous, unified, empirical approach to materiality, reliance, and causation" through empirical econometric methods that made the FOTM theory appealing in the first instance.[84]

Those scholars advocating for the removal of a market efficiency requirement also premised their belief on the notion that "price distortion" was sufficient to establish that the plaintiff was harmed through a material

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79. Fischel, *supra* note 36, at 7.

80. *Id.*

81. Although materiality is an undisputed element of a Rule 10b-5 class action, the Supreme Court has grappled with whether it needs to be established before a class can be certified. In *Amgen Inc. v. Connecticut Retirement Plans & Trust Funds*, 133 S. Ct. 1184, 1191 (2013), the Court held that materiality was not an inquiry that touched on issues of classwide proof and was more appropriately dealt with at the merits stage. About a year later in *Halliburton II* the Court held that, although materiality does not *need* to be established for the class to be certified, defendants can present evidence rebutting the materiality of alleged misrepresentations based on a price impact analysis, seemingly shifting the burden of proof for materiality from the plaintiffs to the defendants in exchange for an expedited review. 134 S. Ct. 2398, 2417 (2014). It is presently unclear what effect this will have on class certification judgments going forward. *See, e.g.*, Merritt B. Fox, *Halliburton II: What It's All About*, 1 J. FIN. REG. 135, 142 (2015).

82. 432 F.3d 261, 269 (3d Cir. 2005) (quoting Oran v. Stafford, 226 F.3d 275, 282 (3d Cir. 2000)).

83. Langevoort, *supra* note 21, at 179.

84. Langevoort, *supra* note 74, at 44 (footnote omitted). In addition to Fischel, *supra* note 36, Langevoort also cites Roger J. Dennis, *Materiality and the Efficient Capital Market Model: A Recipe for the Total Mix*, 25 WM. & MARY L. REV. 373 (1984), as having a significant effect on the holding in *Basic*. Langevoort, *supra* note 74, at 44 nn.39-40.

misrepresentation.[85] Believing that courts were best served focusing on whether the public misstatement was reflected in the market price, these scholars asserted that materiality should be found "[w]henever event study methodology shows that a fraudulent event has had a *statistically significant* effect on the price of a firm's securit[y]."[86] This view presupposed the ease with which courts could analyze the empirical evidence of a stock price reaction to the release of information.

However, "the simplicity was an illusion."[87] Econometric analyses of changes in the price of a security, particularly when conducted by dueling economic experts paid by adversarial parties to a lawsuit, produced entirely divergent results as the rule and not the exception.[88] In his precursor article to *Basic*, Fischel made a bold prediction that likely assuaged the concerns of those Justices hesitant to uphold the FOTM doctrine:

> Moreover, resources spent on securities fraud litigation will be reduced. Because the focal issue of every case will be whether there has been any effect on the market price of the firm's securities, the increased certainty resulting from this objective determination will reduce the amount of litigation. On those occasions when litigation is brought, there will no longer be any need for fact-finding on such issues as what a reasonable investor would have thought important or whether investors were aware of a certain document. *In all probability, therefore, the effect on the market price approach will decrease the overall amount of litigation under rule 10b-5.*[89]

This prediction proved staggeringly inaccurate. Instead of imposing discipline on subjective judicial discretion, Rule 10b-5 claims brought under the FOTM doctrine turned out to be a boon to the securities litigation industry. By 1991, just three years after the ruling in *Basic*, the number of securities class action filings had tripled, and they continued to rise over the following decades.[90] Joseph Grundfest, writing before the portentous second decision in *Halliburton*, noted that securities fraud had become veritable big business. Between 1997 and 2013 over three thousand cases were filed, generating settlements of over $73 billion and "compris[ing] six of the ten largest settlements in class action history."[91]

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85. *See, e.g.*, Macey et al., *supra* note 68, at 1018.
86. *Id.*
87. Langevoort, *supra* note 74, at 44.
88. *See* Langevoort, *supra* note 21, at 179.
89. Fischel, *supra* note 36, at 16 (emphasis added).
90. Langevoort, *supra* note 21, at 179.
91. Grundfest, *supra* note 17, at 308. The article further details how, between 1997 and 2013, plaintiffs' lawyers earned more than $14 billion in fees with defense counsel likely earning something comparable. *Id.* at 309. This represents both a private and public burden; in the years 2002 to 2004, class action securities cases represented nearly half of all class action cases pending in federal court. *Id.*

In response to perceived abuses, including the mechanical filing of lawsuits following price declines, abuse of the discovery rules "with only faint hope that the discovery process might lead eventually to some plausible cause of action,"[92] and improper solicitation of class representatives by plaintiffs' attorneys, Congress passed the Private Securities Litigation Reform Act of 1995 (PSLRA).[93] The PSLRA enacted various procedural safeguards to reduce frivolous lawsuits, many of which addressed the conduct of plaintiffs' attorneys.[94] In addition to these general procedural safeguards, the PSLRA changed substantive pleading requirements for cases brought under Rule 10b-5. Following the enactment of the PSLRA, it was no longer sufficient to simply establish materiality and market efficiency; a moving party was required to demonstrate "loss causation," a statutorily undefined concept.[95]

The Court ultimately provided clarity in *Dura Pharmaceuticals, Inc. v. Broudo*, holding that loss causation signified the plaintiff's burden to establish a direct causal nexus between the defendant's fraud and the economic harm.[96] *Dura* categorically rejected the notion that reliance and loss causation are synonymous concepts, holding instead that both must be established separately.[97] Economic injury would be measured at two different points in time—when the stock was purchased and when the fraudulent misstatement was ultimately disclosed to the market.[98] Viewed in conjunction with the reliance requirement from *Basic*, *Dura* stands for the proposition that "the plaintiff's economic loss is the amount of the original price distortion that remains in the stock until the corrective disclosure, as measured by the market's response to the disclosure of the original misrepresentation."[99] Unsurprisingly, the response by courts to this heightened standard has been "by all accounts a doctrinal and practical mess."[100]

92. H.R. REP. NO. 104-369, at 31 (1995) (Conf. Rep.).

93. Pub. L. No. 104-67, 109 Stat. 737 (1995) (codified as amended in scattered sections of 15 U.S.C.).

94. Gregory Kendall, Comment, *The Artful Dodgers: Securities Fraud, Artful Pleading, and Preemption of State Law Causes of Action Under the Securities Litigation Uniform Standards Act*, 81 U. CIN. L. REV. 657, 660 (2012).

95. Fisch, *supra* note 30, at 914.

96. *See* 544 U.S. 336, 345-46 (2005); Fisch, *supra* note 30, at 915.

97. *Dura*, 544 U.S. at 346; *see also* Fisch, *supra* note 30, at 915.

98. Fisch, *supra* note 30, at 915. Under the logic of *Dura*, a plaintiff who purchased an overvalued share but who is able to offload the share before the market discovered the fraud suffered no injury. Any decrease in the price over this period resulted from factors unrelated to the fraud. *Id.*

99. *Id.*

100. Langevoort, *supra* note 74, at 45.

3. Damages

A remaining practical concern, and one which figures prominently in any discussion of securities litigation practice, pertains to the calculation of damages. In none of its twenty-eight opinions interpreting the scope of section 10(b) class actions has the Supreme Court opined on the question of after-market damages.[101] Lacking explicit guidance, most lower courts have adopted the "out-of-pocket" damages[102] standard set forth in *Affiliated Ute Citizens v. United States*.[103] Under *Affiliated Ute*, each purchaser of a security is entitled to the difference between the price paid for the security and the price it would have traded at had there been no fraudulent misrepresentation or omission.[104] In addition to the inherent difficulties of determining the "but-for" trading price (generally done through the use of an event study, as described in Part III below), an aggregate damage calculation is contingent on an estimation of the precise number of shares entitled to recovery and statistical adjustments for the frequency with which shares changed hands.[105] Although some courts were under the impression that computing individual damages would be "virtually a mechanical task,"[106] these determinations require complicated statistical calculations that courts and finders of fact are largely unqualified to evaluate.

Against this backdrop of complexity, it is all the more disconcerting that the Supreme Court has yet to provide clarity. As Grundfest notes, "[T]his entire statistical methodology governing a multi-billion dollar litigation market, in which subtle differences in econometric technique can have significant impact on plaintiff recoveries and defendant exposures, has evolved without any Supreme Court oversight."[107] One reason for the dearth of judicial influence on the subject is certainly the overall infrequency with which cases proceed to trial. Given the immense liability attached to securities class actions, the potential inability of the jury to understand complex econometric disputes, and the high cost of litigation, settlement pressure is immense. Since Congress

101. Grundfest, *supra* note 17, at 310.

102. *Id.* at 364-65.

103. *See* 406 U.S. 128, 155 (1972).

104. Janet Cooper Alexander, *The Value of Bad News in Securities Class Actions*, 41 UCLA L. REV. 1421, 1428-29 (1994).

105. *Id.* at 1432. Alexander notes that "the trades of 'ins-and-outs' must be estimated through a statistical model. Building such a model depends on an assumption about the statistical probability that any particular share will trade on a given day." *Id.* at 1459. Alexander further posits that the model most commonly used by plaintiffs, the proportional trading model, assumes that there is an equal chance of trading for each share and may inflate the total class damage amount by one hundred percent or more. *Id.* at 1459-60, 1462.

106. Blackie v. Barrack, 524 F.2d 891, 905 (9th Cir. 1975).

107. Grundfest, *supra* note 17, at 365.

passed the PSLRA, only twenty cases have gone to trial, and only fourteen of those reached a verdict.[108] However, even if settlement pressure induces parties to resolve disputes privately, it does not follow that judicial practice is best served by allowing parties to resolve damage disputes among themselves.

Janet Cooper Alexander notes that the issue of computing damages may actually be *more* consequential when a case is settled out of court.[109] Whereas trials devote significant attention to addressing liability, settlement discussions are largely centered on the amount of compensation.[110] When overall uncertainty exists surrounding the proper method of computing damages, settlement discussions "may be impeded or distorted."[111] Even with general agreement regarding the proper model of damages, calculations made by opposing parties are often orders of magnitude apart. The lack of established judicial precedent leaves the parties uninformed on implementing a standard for measuring damages and constructing favorable negotiating positions anchored to this standard.[112] In the absence of Supreme Court guidance, the parties and their respective expert witnesses act with minimal supervision in crafting damage calculations used for settlement, with distortionary impacts on private adjudication.

In sum, as a result of decades of federal court adjudication, more than two dozen Supreme Court decisions, and intermittent legislative guidance, the standard for a private right of action under section 10(b) now contains a number of discrete requirements in addition to traditional common law standards. Plaintiffs must establish that the market for the security in question is semi-strong form efficient, normally through the use of the *Cammer* factors, to obtain the presumption of reliance established in *Basic*. Additionally, they are required to demonstrate that the purportedly false and misleading statements changed the "total mix" of information available to the market and that the misrepresentation had a direct causal connection to their ultimate claim of harm, both of which are now largely empirical determinations. Finally, after establishing liability, plaintiffs must put forward a defensible

108. RENZO COMOLLI & SVETLANA STARYKH, NERA ECON. CONSULTING, RECENT TRENDS IN SECURITIES CLASS ACTION LITIGATION: 2013 FULL-YEAR REVIEW 36 (2014), http://www.nera.com/content/dam/nera/publications/2014/PUB_Year_End_Trends_1.2014.pdf.

109. Alexander, *supra* note 104, at 1422.

110. *Id.*

111. *Id.* at 1423. Alexander notes that with divergent damage estimates, parties may find it hard to reach a "zone of agreement" in which a settlement can occur. *Id.* These differences will compound the "psychological barriers" that already impede the efficient resolution of a negotiation. *Id.*

112. *Id.* at 1424.

calculation of classwide damages through the use of an event study and statistical estimates of trading activity.

II. Role of Expert Testimony in Securities Fraud Litigation

Following the doctrinal shift from traditional standards of materiality and reliance to a market-oriented norm, expert testimony took on added significance. As described in the preceding Part, there are four objective areas of dispute in the prosecution of class action lawsuits under Rule 10b-5: reliance, materiality, loss causation, and damages. Each of these considerations is critically dependent on the provision of a reliable event study by a qualified expert.[113]

An event study is a statistical analysis of the effect of an "event" on the price of a security. Determinations are made by comparing the actual return to the return predicted by the contemporaneous change in a benchmark index of comparable stocks and the security's historical comovement with the market.[114] At the time the FOTM doctrine was established, it was generally assumed that event studies were robust to methodological choice.[115] More recent scholarship, however, has recognized the inherent ambiguity and challenges associated with event study design.[116]

A. Overview of an Event Study

The event study is a tool appropriated from the financial economics literature, in which it is commonly used to assess the impact of a general type of event over a large cross-section of securities.[117] The use of event studies in litigation is appealing because, in an efficient market, the price of a security will immediately reflect the effect of an event.[118] Courts have consequently

113. *See* Kaufman & Wunderlich, *supra* note 1, at 187.

114. *See* Alexander, *supra* note 104, at 1433.

115. *See, e.g.*, Macey et al., *supra* note 68, at 1030 ("[R]esearchers have shown that the findings of event studies using different methodologies are robust in a wide variety of situations. That the findings of event studies using any of a number of methodologies are very similar is especially true when testing for materiality in a fraud-on-the-market theory case—the effect on stock returns of an important piece of news released over a short period of time." (footnote omitted)).

116. *See, e.g.*, Fisch, *supra* note 30, at 919 ("Although event studies are used extensively, they are imperfect tools for measuring the effect of a disclosure on stock prices. . . . [T]heir application presents a number of methodological challenges.").

117. *See, e.g.*, Eugene F. Fama et al., *The Adjustment of Stock Prices to New Information*, 10 INT'L ECON. REV. 1, 3 (1969).

118. A. Craig MacKinlay, *Event Studies in Economics and Finance*, 35 J. ECON. LITERATURE 13, 13 (1997).

used event studies to analyze a range of disputes, including mergers and acquisitions, earnings announcements, issuance of debt and equity securities, and the effect of regulatory changes.[119] Their ubiquity in litigation has led some to declare that, "[a]s large a role as event studies play in empirical financial economics and policy analysis, their importance in litigation (e.g., under SEC Rule 10b-5), may be even greater."[120] However, while many of the statistical assumptions inherent to event studies are of minor concern when applied across time and across securities, their significance is magnified when applied to one security for only a select number of events.[121]

Although there is considerable academic debate regarding the statistical methodology used in event studies, the "general flow of analysis" is reasonably established.[122] There are three practical conditions necessary to properly conduct a useful event study: (1) a return series covering the event at issue is available, (2) the stock trades frequently enough for each return to cover only one day (or at most a few days), and (3) the parties can confidently establish the dates on which the event in question occurred.[123] Once established, there are three basic facets of conducting an event study: (1) defining the "event window," (2) calculating the abnormal returns of the stock over the event window, and (3) testing for statistical significance of the abnormal return.[124]

The event window is the period over which the effect of the event on the security will be analyzed. Because event studies are premised on the efficient market hypothesis, the presumption is that the stock price will quickly reflect new information when released to the market. Consequently, event windows used for litigation are typically quite short and may cover only the one-day trading period following the event.[125] If the exact time that the information was released to the market is uncertain, or if the analyst has reason to believe that the information was not quickly absorbed into the stock price, courts may allow for longer event windows.[126] However, a longer event window can

119. *Id.*

120. Jonah B. Gelbach et al., *Valid Inference in Single-Firm, Single-Event Studies*, 15 AM. L. & ECON. REV. 495, 499 (2013).

121. *See* Fisch, *supra* note 30, at 920.

122. MacKinlay, *supra* note 118, at 14.

123. *See* M. Laurentius Marais & Katherine Schipper, *Event Study Methods: Detecting and Measuring the Security Price Effects of Disclosures and Interventions (New)*, *in* LITIGATION SERVICES HANDBOOK: THE ROLE OF THE FINANCIAL EXPERT 17A.1, 17A.9 (Roman L. Weil et al. eds., 3d ed. Supp. 2005).

124. *See* Mark L. Mitchell & Jeffry M. Netter, *The Role of Financial Economics in Securities Fraud Cases: Applications at the Securities and Exchange Commission*, 49 BUS. LAW. 545, 557-58 (1994).

125. *See id.* at 558.

126. *See id.* at 558-59.

compromise the ability of the event study to identify abnormal performance.[127]

After defining the event window, one must isolate the portion of the security return attributable to the news. The event study is primarily a method of determining whether estimated event effects fall outside the range that would be expected given the normal variation of stock returns, thereby allowing the remaining variation to be attributed to firm-specific factors. The first practical decision is whether to calculate the return series in gross[128] or logarithmic (log) form.[129] Although financial economists prefer the statistical properties of log series, this decision is likely to have little practical effect.[130]

The more significant determination is in modeling normal performance, or the "expected return." Model variants are largely divided into two categories: "statistical" and "economic."[131] Statistical models rely solely on the empirical behavior of asset returns, while economic models rely additionally on theories of individual investment behavior.[132] Because economic models impose additional statistical assumptions without offering significant practical advantage, economists prefer statistical models.[133] While statistical models do not rely on the validity of an underlying economic argument, they still assume that asset returns are jointly multivariate-normal and independent and identically distributed across time.[134]

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127. *See* David I. Tabak & Frederick C. Dunbar, *Materiality and Magnitude: Event Studies in the Courtroom* 8 (Nat'l Econ. Research Assocs., Working Paper No. 34, 1999).

128. These are arithmetically derived as the change in the stock price during the period, plus any dividends paid, divided by the previous closing price. The gross return can thus be expressed as $R_t = \frac{(P_t - P_{t-1}) + D_t}{P_{t-1}}$.

129. Researchers in financial economics generally prefer logarithmic return series, which are continuously compounded and expressed as the natural logarithm of one plus the gross return, or $LR_t = ln\left(\frac{P_t + D_t}{P_{t-1}}\right)$. The log transformation causes the return distribution to be closer to normal, which is the basis for the inference tests used to determine statistical significance. The assumption that continuously compounded single-day returns are independent and identically distributed has been called "the workhorse of the financial asset pricing literature." JOHN Y. CAMPBELL ET AL., THE ECONOMETRICS OF FINANCIAL MARKETS 15-16 (1997). Additionally, logarithmic returns have been found to produce better test specifications than tests based on arithmetic returns in event studies. *See* Charles J. Corrado & Cameron Truong, *Conducting Event Studies with Asia-Pacific Security Market Data*, 16 PAC.-BASIN FIN. J. 493, 509 (2008).

130. *See* Mitchell & Netter, *supra* note 124, at 560 n.96.

131. MacKinlay, *supra* note 118, at 17.

132. *Id.*

133. *See* CAMPBELL ET AL., *supra* note 129, at 156-57.

134. MacKinlay, *supra* note 118, at 17. This assumption, generally assumed to be of only minor significance, has been called into question and is the overriding concern regarding methodological choice in the finance literature. *See infra* Part III.

Part III below details the academic literature on methodological choice. For the purpose of describing the use of event studies in expert testimony, it is an adequate generalization that a "market model" event study that estimates predicted returns through the use of an ordinary least squares (OLS) regression is the standard adopted by most courts.[135] Using OLS, the analyst computes the historical relationship between the return on the asset and the return on the market by regressing the former on a representative market index and/or an index constructed to represent the return on companies within the same industry group.[136] The period over which this regression analysis is computed, the "estimation window" or "control period," is typically placed before the beginning of the "class period"[137] so that the distortionary effect of the allegedly misrepresented information will not influence the estimated historical relationship.[138] In this regard, conducting an event study for securities litigation departs from its application in the academic literature. In other environments it may be necessary to use postevent data to estimate the market model if there is a suspected time-varying change in the correlation between the stock and market returns.[139] However, a class action suit alleging fraudulent misrepresentations presupposes a discernable change in the pattern of stock returns, and using postdisclosure data would risk having the event impact the generated expected returns.

As noted above, the analyst must choose a market index to use in estimating the regression equation between the stock and the market returns. Event studies often use a market model with a single market index, such as the S&P 500.[140] These studies can also be augmented to include the return on a peer group, which frequently consists of firms in the same Standard Industrial

135. *See, e.g.*, Tabak & Dunbar, *supra* note 127, at 8 n.19 ("While some crude event studies are performed without adjusting for market effects, the literature nearly uniformly argues that a market adjustment is desirable. Moreover, there is relevant case law, such as *In Re Executive Telecard Ltd. Securities Litigation*, which states that in measuring stock price declines, one must eliminate 'that portion of the price decline that is the result of forces unrelated to the wrong.'" (quoting *In re* Exec. Telecard Ltd. Sec. Litig., 979 F. Supp. 1021, 1025 (S.D.N.Y. 1997))); *see also* Macey et al., *supra* note 68, at 1034-35 (claiming that the language of section 11 of the Securities Act implies a need for market adjustment); Mitchell & Netter, *supra* note 124, at 567 (describing the basic method for adjustment as the market model regression).

136. *See* Tabak & Dunbar, *supra* note 127, at 8-10.

137. The class period is the time between the first alleged misrepresentation and when the fraud is disclosed to the market, and it is the date range analyzed through expert testimony. Only purchasers of securities during the class period are eligible for recovery. *See* Eric Helland, *Reputational Penalties and the Merits of Class-Action Securities Litigation*, 49 J.L. & ECON. 365, 370 (2006).

138. *See* Tabak & Dunbar, *supra* note 127, at 9 & n.21.

139. *See* MacKinlay, *supra* note 118, at 20.

140. *See* Marais & Schipper, *supra* note 123, at 17A.11.

Classification (SIC) code.[141] Once the analyst has chosen the relevant dependent variable(s), an OLS regression is conducted of the daily security returns on the daily market returns over the estimation period, assuming the return on the stock is a function of the return on the market and an error component representing firm-specific effects. A representative single-factor market model is of the form:

$$security_t = \alpha + \beta market_t + \varepsilon_t$$

The estimated equation calculates a constant market-model intercept for the security, α (alpha), and a coefficient measure of the sensitivity of the firm's stock to the broader market, β (market beta). In practical terms, alpha represents the expected return of the security when the market return is zero, while the market beta represents the tendency of the security to react to a given change in the market. The daily predicted return is calculated as the sum of the market-model intercept term and the product of the market beta and contemporaneous return of the market index. For example, if a firm has an alpha of 0 and a market beta of 1.5, when the market index return is -1% the expected one-day return will be -1.5% (0% + 1.5 * -1% = -1.5%). The unexpected variation in the stock return—in the economic literature, the "abnormal return"—is simply the difference between the observed daily return of the stock and the calculated predicted return. This is mathematically identical to the daily residual, ε_t, computed through the regression equation.

The final stage of an event study is to analyze the statistical significance of the daily abnormal return. Because security prices fluctuate naturally, it is necessary to calculate the level of confidence that the event-induced abnormal return is not zero. This is accomplished by comparing the ratio of the estimated daily abnormal return, ε_t, to the standard deviation (a measure of the dispersion of a variable around its mean) of the residuals from the regression equation.[142] The resulting ratio is commonly referred to as a "*t*-statistic"[143] and

141. *Id.* at 17A.11-12. As the authors note, the appropriateness of including additional factors often depends on context and is largely an empirical question. *Id.* at 17A.12.

142. In practice, the root mean squared error, calculated as the square root of the sum of the squared residuals, is used as the divisor in the ratio. The estimated residual variance from the regression model and the standard deviation of the net-of-market returns over the estimation period are "virtually identical," with the root mean squared error being slightly more precise in accounting for firm-specific variation to the market. Mitchell & Netter, *supra* note 124, at 569 n.113; *see also* Pamela P. Peterson, *Event Studies: A Review of Issues and Methodology*, Q.J. BUS. & ECON., Summer 1989, at 43 (1989) (noting that the standard error of the estimated regression is used in standardizing abnormal returns when simple regression analysis is used).

143. The ratio of an estimate to its standard error is called the *t*-statistic, and because, in this instance, the statistic references the residual value of the regression, it is also known as the "studentized residual." *See* Gelbach et al., *supra* note 120, at 502.

can be compared to the probability density of the student *t* distribution[144]: a normally distributed variable will have 95% of its observations fall within approximately two standard deviations of its mean. Thus, by assuming that the returns adhere to normality, an event abnormal return will be found to have *not* occurred by chance when the absolute value of the *t*-statistic is greater than or equal to roughly 1.96.[145]

It should be noted that the ease of this comparison is contingent on the tested variable being distributed normally; if the returns come from a different distribution, inferences based on the probability density function will be inaccurate. Historically, it was assumed that the normal distribution was an accurate enough description of daily stock returns.[146] Even those studies acknowledging the non-normality of daily returns found it to be of little impact when interpreting the results of an event study.[147] However, recent literature has called this assumption into question, as demonstrated in Part III below.

B. Using an Event Study to Analyze Rule 10b-5 Requirements

Armed with the event study results, an expert witness is able to address reliance, materiality, loss causation, and damages. With regard to reliance, defendants often dispute whether the market for the company's security is semi-strong form efficient, a necessary predicate for *Basic*'s presumption of reliance.[148] For smaller, off-exchange securities, defense experts may be able to demonstrate low trading volume, a comparatively large bid-ask spread,[149] or

144. *See* Moultrie v. Martin, 690 F.2d 1078, 1084 n.10 (4th Cir. 1982) ("The student's t distribution, like the binomial distribution . . . is represented by a bell shaped curve. When the sample size is small, the student's t curve is flatter in the middle and plumper in the tails.").

145. This critical value is based on a two-tailed test, which compares the residual to the probability distribution in the two furthest ends of the bell curve. In theory, if you know the direction of the expected return, the proper comparison should be to only one end of the distribution (also known as a one-tailed test). For the sake of accuracy, I will use a two-tailed test when testing Type I errors—which by construction can be both positive and negative—whereas Type II power analyses will only test against the tail representing the sign of the imputed value.

146. *See, e.g.*, Mitchell & Netter, *supra* note 124, at 563.

147. *See, e.g.*, Macey et al., *supra* note 68, at 1039 n.67.

148. As mentioned in Part I.C.3 above, plaintiffs must establish that the market for the security in question is semi-strong form efficient to obtain the presumption of reliance established in *Basic*.

149. The bid-ask spread represents the difference between the price at which investors will buy a stock and the price at which holders of the security are willing to sell. Some courts have found a comparatively large bid-ask spread to be indicative of an inefficient market because "the stock is too expensive to trade." *See, e.g.*, Krogman v. Sterritt, 202 F.R.D. 467, 478 (N.D. Tex. 2001).

other objective bright-line standards in disputing efficiency. The more contentious battle, particularly for blue-chip stocks, lies in testing the fifth *Cammer* factor: whether a demonstrable relationship exists between the release of unexpected material information and a change in the price of the security. If it can be shown through an event study that the stock did not react in a statistically significant manner to value-relevant information, the court may find that the efficiency requirement is not satisfied and deny class certification.

The modern interpretation of materiality constituting information that affects investment decisions lends itself to empirical testing. As Frederick Dunbar and Dana Heller note:

> [O]ne can ask the economic question of how a change in investors' decisions to trade at a given price could be observed. The straightforward answer is that if the information would cause more investors to want to buy at a particular price, the previous supply-and-demand equilibrium would be upset and the price would have to rise until the demand for the stock once again equaled its supply. This, of course, says that materially positive news causes a stock's price to rise.[150]

From an economic perspective, it can only be confirmed that the price of the security reacted to news if the abnormal return is statistically significant; absent this determination, a price change could instead be merely an artifact of the normal daily fluctuation. The approach to materiality accepted by courts is therefore framed objectively and identified by an event study.[151]

Loss causation follows a similar pattern. Post-*Dura*, plaintiffs are required to demonstrate the causal link between the alleged harm and the purportedly fraudulent statements or actions by the company. Event studies are the cleanest mechanism available to establish this connection, and they can be used to demonstrate both that an economic loss occurred and that the loss can be proximately connected to the underlying misrepresentations.[152]

Finally, an event study is an integral component in computing damages. In order to determine class damages under the out-of-pocket measure stipulated in *Affiliated Ute*, the expert must establish the daily price at which the security would have traded had there been no fraudulent representations, also called a "value line."[153] For individual trades, damages can be calculated as the difference between the price paid and the value line, multiplied by the number

150. Dunbar & Heller, *supra* note 14, at 468.

151. *See, e.g.*, Gen. Elec. Co. v. Jackson, 595 F. Supp. 2d 8, 23-24 (D.D.C. 2009) (declaring that a regression-based event study was probative of the effect of environmental noncompliance letters on stock prices).

152. Kaufman & Wunderlich, *supra* note 1, at 198.

153. *See* Bradford Cornell & R. Gregory Morgan, *Using Finance Theory to Measure Damages in Fraud on the Market Cases*, 37 UCLA L. REV. 883, 885 (1990) (quoting Green v. Occidental Petrol. Corp., 541 F.2d 1335, 1344 (9th Cir. 1976) (Sneed, J., concurring in part and concurring in the result in part)).

of shares purchased.[154] In the aggregate, class damages can be approximated by the daily disparity between the share price and the value line, multiplied by the volume of shares traded.[155] Because calculating the value line with reference to fundamental company value, earnings data, or analyst expectations involves inherently subjective components, the task of calculating the value line has been delegated to the event study.[156]

As recognized in Part I.3.C above, the proper approach to the damages portion of a securities fraud suit is unsettled, which has led to substantial differences in methodology. Although an exhaustive discussion of the competing methods and comparative benefits of each approach is beyond the scope of this Note,[157] suffice it to say that if the opposing experts disagree on the event study model used for purposes of reliance or materiality, their measures of damages will likely differ by an order of magnitude. This divergence will have significant effects on the parties' ability to reach a settlement agreement that is in both sides' interest. Given the critical reliance on event studies at each stage of the securities fraud process, there exists a surprising paucity of studies empirically analyzing the performance of different event study models.

III. Literature Review on Event Study Models

The event study is "one of the most frequently used analytical tools" in corporate finance research.[158] Before the advent of the modern event study in 1969, there was little empirical evidence of the central issues of financial economics, whereas "[n]ow we are overwhelmed with results, mostly from event studies."[159] The ability to isolate the impact of a broad range of corporate events occurring in capital markets led to a dramatic increase in published articles using the event study technique; Kothari and Warner report that

154. Alexander, *supra* note 104, at 1429.

155. *Id.* at 1429-30.

156. *See* Cornell & Morgan, *supra* note 153, at 888.

157. For excellent reviews of the issues associated with measuring damages in securities class action lawsuits, see Robert A. Alessi, *The Emerging Judicial Hostility to the Typical Damages Model Employed by Plaintiffs in Securities Class Action Lawsuits*, 56 BUS. LAW. 483 (2001); Janet Cooper Alexander, *Rethinking Damages in Securities Class Actions*, 48 STAN. L. REV. 1487 (1996); Michael Barclay & Frank C. Torchio, *A Comparison of Trading Models Used for Calculating Aggregate Damages in Securities Litigation*, LAW & CONTEMP. PROBS., Spring/Summer 2001, at 105; and Edward I. George & William E. Wecker, *Estimating Damages in a Class Action Litigation*, 3 J. BUS. & ECON. STAT. 132 (1985).

158. Peterson, *supra* note 142, at 36.

159. Eugene F. Fama, *Efficient Capital Markets: II*, 46 J. FIN. 1575, 1600 (1991).

between 1974 and 2000, 565 papers containing event studies were published in five finance journals alone.[160]

The event study method as commonly used was established in an influential 1969 paper by Eugene Fama, Lawrence Fisher, Michael Jensen, and Richard Roll.[161] In examining the effect of stock split announcements on the value of common equity, the authors established the textbook table layout that is still the basis of the standard event study.[162] Although the structural format of an event study has remained stable, significant intellectual resources have been devoted to researching more sophisticated statistical modeling techniques and more accurate means of adjusting the measure of statistical significance to ensure the validity of inferences drawn from event studies.[163]

Beginning in the 1980s, a parallel literature developed analyzing the comparative ability of the various preexisting statistical models to detect abnormal performance. A pair of seminal companion articles written by Stephen Brown and Jerold Warner analyzed the specification properties of these models and their ability to detect abnormal performance using monthly[164] and daily[165] data. Brown and Warner's 1985 paper, which "has since come to eponymously define the genre,"[166] found that event studies presented few practical difficulties when using daily data.[167] Although daily returns clearly departed from normality, methodologies based on OLS market models were "well-specified under a variety of conditions."[168] The academic community was generally convinced that event studies represented an empirically valid method of testing financial hypotheses, even given the strict assumptions generally required in parametric hypothesis testing.

Dozens of papers have been published testing the properties of competing event study methods.[169] These studies analyze two primary characteristics: how frequently the statistical test rejects the null hypothesis of no abnormal price performance, and how frequently the null hypothesis is rejected in the presence of a known abnormal return.[170] The first inquiry, often known as the

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160. Kothari & Warner, *supra* note 2, at 6.

161. Fama et al., *supra* note 117.

162. *See* Kothari & Warner, *supra* note 2, at 8.

163. *See id.*

164. Brown & Warner, *supra* note 3.

165. Brown & Warner, *supra* note 4.

166. Charles J. Corrado, *Event Studies: A Methodology Review*, 51 ACCT. & FIN. 207, 213 (2011).

167. Brown & Warner, *supra* note 4, at 25.

168. *Id.*

169. Kothari & Warner, *supra* note 2, at 5.

170. *See* John J. Binder, *The Event Study Methodology Since 1969*, 11 REV. QUANTITATIVE FIN. & ACCT. 111, 120 (1998).

"analysis of specification," tests whether the Type I error rate (i.e., when the null hypothesis of no abnormal performance is falsely rejected) approaches the error rate of the assumed size of the test.[171] The second inquiry, called the "analysis of power," tests the ability of a model to detect abnormal price performance when it exists; the failure to do so is known as a Type II error.[172] When comparing tests that are well specified, the test with higher power is preferred. Using pseudo-simulations, "artificial" abnormal performance is imputed into actual stock returns, and the ability of different models to detect statistically significant abnormal returns is analyzed.[173]

The initial comparative performance studies evaluated the mean-adjusted return model, the market-adjusted return model, and the OLS market model. The mean-adjusted model calculates abnormal returns by simply subtracting the average return of the stock during the estimation period and comparing each out-of-sample daily abnormal return to the standard deviation of the average.[174] This method does not explicitly control for the idiosyncratic risk of the stock or the contemporaneous return on the market. The market-adjusted return model subtracts the return on the market from the daily return and compares the difference in the event period to its mean and standard deviation in the control period.[175] As detailed in Part II above, the market model approach calculates abnormal performance by using pre-event period returns and an OLS regression. This approach controls for both the risk of the stock (as measured by its market beta) and the simultaneous returns on the market.[176]

The initial Brown and Warner studies were notable for finding that modeling choice did not have a material impact on the performance of event studies.[177] Although the authors found that daily data presented few difficulties for properly conducting an event study, they did acknowledge that an increase in security variance could lead to too many rejections of the null hypothesis that the average excess return is zero.[178]

Later empirical studies questioned the findings of these initial counterintuitive results. Ramesh Chandra, Shane Moriarity, and G. Lee Willinger demonstrated that the comparability in performance of the mean-

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171. *See* Kothari & Warner, *supra* note 2, at 12. The assumed size of the test corresponds to the confidence level used in testing statistical significance: a test based on a 95% confidence level should result in tests results finding statistical significance 5% of the time.

172. *See id.*

173. *See, e.g.*, Brown & Warner, *supra* note 4, at 8-16.

174. *See id.* at 6-7.

175. *See id.* at 7.

176. *See id.*

177. Brown & Warner, *supra* note 3, at 249; Brown & Warner, *supra* note 4, at 12.

178. Brown & Warner, *supra* note 4, at 23.

adjusted and market-adjusted return models was largely a statistical artifact of model implementation.[179] More generally, many academics challenged the notion that violations of normality in the underlying returns of the security were irrelevant to the performance of the model. Subsequent research verified the Brown and Warner conclusion that abnormal returns are not normally distributed, but it instead found this violation to cause significant problems of inference, leading to both under- and overrejection of the null.[180] For "outlier-prone data," prevalent in financial markets, the true Type I error rate will be larger than that associated with particular asymptotic values, with greater discrepancies found in stock returns with higher levels of kurtosis.[181]

Recognizing the limitations of standard inference tests in the presence of normality violations, scholars searched for alternative statistical models that would be robust to the empirical distribution of abnormal returns. Some have proposed using nonparametric tests of abnormal performance, which make no assumption about the probability distribution of the variables. The most successful of the nonparametric tests have been the rank and sign tests. More applicable to the context of single-firm, single-event studies, the nonparametric rank test transforms the distribution of the abnormal returns into a uniform distribution across rank values irrespective of the original distribution.[182] An alternative method is to normalize the conventional *t*-statistics from the market model regression with bootstrap resampling.[183] Evidence on the performance of bootstrap methods has been mixed and varies with the bootstrap resampling's application; as a result, it has not enjoyed popular support in the event study literature.[184] The overall conclusion from these articles is that alternative event study methods, whether parametric or

179. *See* Ramesh Chandra et al., *A Reexamination of the Power of Alternative Return-Generating Models and the Effect of Accounting for Cross-Sectional Dependencies in Event Studies*, 28 J. ACCT. RES. 398, 400 (1990).

180. *See, e.g.*, George S. Ford & Audrey D. Kline, Event Studies for Merger Analysis: An Evaluation of the Effects of Non-Normality on Hypothesis Testing (Aug. 2006) (unpublished manuscript), http://ssrn.com/abstract=925953.

181. Scott E. Hein & Peter Westfall, *Improving Tests of Abnormal Returns by Bootstrapping the Multivariate Regression Model with Event Parameters*, 2 J. FIN. ECONOMETRICS 451, 456 (2004). Kurtosis is formally defined as "the standardized fourth population moment about the mean" and is used to describe "the type and magnitude of departures from normality." Lawrence T. DeCarlo, *On the Meaning and Use of Kurtosis*, 2 PSYCHOL. METHODS 292, 292, 302 (1997).

182. Charles J. Corrado, *A Nonparametric Test for Abnormal Security-Price Performance in Event Studies*, 23 J. FIN. ECON. 385, 388 (1989).

183. *See, e.g.*, Lisa A. Kramer, *Alternative Methods for Robust Analysis in Event Study Applications*, *in* 8 ADVANCES IN INVESTMENT ANALYSIS AND PORTFOLIO MANAGEMENT 109 (Cheng-Few Lee ed., 2001).

184. Corrado, *supra* note 166, at 216.

variations based on empirical resampling of the abnormal return distribution, should be implemented when the data are distributed non-normally.

Scholars have recently scrutinized the application of event studies in particular relation to their use in litigation. Corrado notes that single-security event studies are rarely reviewed in academic literature but are routinely used in legal proceedings.[185] He advises legal practitioners to use a simple modification of the standard event study approach, which "merely involves counting the number of returns from the control period that are larger or smaller than the event date return."[186] In reviewing event studies as applied to Rule 10b-5 securities fraud cases, Gelbach et al. propose a similar modified event study procedure called the "SQ test."[187] Like the test endorsed by Corrado, the SQ test involves ranking the abnormal returns from the market model regression and testing whether the event-date abnormal return is larger (or smaller) than the abnormal return quantile corresponding to a given confidence level.[188] Refuting the doctrinal reliance on the central limit theorem,[189] the authors prove that the large-sample behavior of the *t*-statistic will be normal only if the abnormal return distribution is itself normal.[190] As a result, standard parametric approaches may yield biased results depending on the size of the event effect and the deviation of the empirical return values from the normal distribution.[191] Using a dataset containing the returns for all securities in the Center for Research in Security Performance's (CRSP) database from 2000 to 2007, the authors find evidence of substantial bias against finding statistically significant abnormal returns.[192]

There has also been increased scholarly interest in the effect of changes in volatility on the inference properties of event studies. Brown and Warner's initial comparative review using daily data noted that an increase in variance would lead to too many rejections of the null hypothesis of no abnormal performance.[193] Aktas, De Bodt, and Cousin note that idiosyncratic volatility is not constant through time and that individual stocks have become more

185. *Id.* at 209.

186. *Id.* at 211.

187. Gelbach et al., *supra* note 120, at 497.

188. *See id.*

189. The central limit theorem is a common, convenient justification for assuming normality in random data. *Id.* at 510.

190. *Id.* at 510-11.

191. *Id.* at 525.

192. *See id.* at 513.

193. Brown & Warner, *supra* note 4, at 23.

volatile over recent decades.[194] Although they do not find an effect on specification tests, the power of event studies to detect abnormal performance varies with idiosyncratic volatility.[195] The authors ultimately conclude that "there is no practical solution to this problem" outside of "increas[ing] the sample size to compensate for the increase in . . . volatility."[196]

IV. Data and Methodology

A. The Financial Crisis and Return Series Data

This Note attempts to answer the questions whether the standard OLS market model or the alternative model proposed by Gelbach perform adequately when used to analyze return series during the financial crisis of 2007-2008, and if not, whether there are readily available alternatives that can be substituted by courts. As previously discussed in Part III, large increases in variance can result in misspecification of the event study model. Given the speed and depth of the shift in market volatility associated with the recent crisis, a reasonable a priori hypothesis is that unadjusted models will overreject the null hypothesis of no abnormal return. If not sufficiently appreciated by courts, this overrejection will lead to a finding of significant event effects where none exist.

An exhaustive review of the events precipitating the collapse of the financial markets is unnecessary for this analysis. However, for the purpose of explaining the date range used to compare results across models, I briefly explain some of the larger events that have been viewed as guideposts to understanding the financial crisis. The decision by BNP Paribas, a French investment bank, to suspend redemptions in three investment funds is considered by many as the "ringing of the bell" marking the beginning of the 2007 liquidity crisis.[197] On August 9, 2007, BNP withheld redemptions following massive reductions in fund value, while also stating that "the complete evaporation of liquidity in certain market segments of the US securitization market has made it impossible to value certain assets fairly regardless of their quality or credit rating."[198] The resulting panic in the

194. Nihat Aktas et al., *Idiosyncratic Volatility Change and Event Study Tests*, 30 FINANCE, no. 2, 2009, at 31, 33.

195. *Id.* at 35.

196. *Id.*

197. NAT'L COMM'N ON THE CAUSES OF THE FIN. & ECON. CRISIS IN THE U.S., THE FINANCIAL CRISIS INQUIRY REPORT 250-51 (2011) [hereinafter FCIC REPORT].

198. *Id.* (quoting Press Release, BNP Paribas, BNP Paribas Investment Partners Temporaly [sic] Suspends the Calculation of the Net Asset Value of the Following Funds: Parvest Dynamic Abs, BNP Paribas Abs Euribor and BNP Paribas Abs Eonia (Aug. 9, 2007),
footnote continued on next page

commercial paper and repurchase agreement markets led to swift government action, with the Federal Reserve committing to provide liquidity in order to facilitate the functioning of financial markets.[199]

The failure of Lehman Brothers on September 15, 2008 led to a run on money market funds and a spike in the commercial paper market.[200] Afterwards, even large industrial corporations, removed from the financial instruments at the heart of the crisis, found it difficult to sell their commercial paper.[201] Markets remained in turmoil following Lehman's collapse, culminating in the Federal Reserve bailout of AIG, the Reserve Primary Fund "breaking the buck,"[202] and congressional approval of the $700 billion Troubled Asset Relief Program.[203] The precise period when the crisis in the financial markets officially abated is less clear. For the purposes of this Note, I chose February 23, 2009, when the Treasury Department, Federal Deposit Insurance Corporation, Office of the Comptroller of the Currency, Office of Thrift Supervision, and the Federal Reserve Board issued a joint statement that "the U.S. government stands firmly behind the banking system, and that the government will ensure that banks have the capital and liquidity they need to provide the credit necessary to restore economic growth."[204]

The turmoil in the equity market can be viewed schematically by analyzing the value of the VIX Index, a barometer of equity market volatility produced by the Chicago Board Options Exchange,[205] over the crisis period. The VIX Index, often referred to as the market's "fear gauge," is designed to measure investor consensus of the thirty-day expected stock market volatility.[206] For empirical analysis, given that a stock price represents a claim on prospective value, the VIX Index is preferred as a volatility proxy to alternatives that only capture historical patterns. The Index value is calculated through the implied volatility of near- and next-term put and call options with

http://www.bnpparibas.com/en/news/press-release/bnp-paribas-investment-partners-temporaly-suspends-calculation-net-asset-value-fo).

199. *Id.* at 252.

200. *See id.* at 339.

201. *See id.*

202. "Breaking the buck" is a euphemism used for the net asset value of a money market fund falls below the one-dollar mark.

203. *The Financial Crisis: Full Timeline*, FED. RESERVE BANK ST. LOUIS, https://www.stlouisfed.org/financial-crisis/full-timeline (last visited May 5, 2016).

204. *Id.*

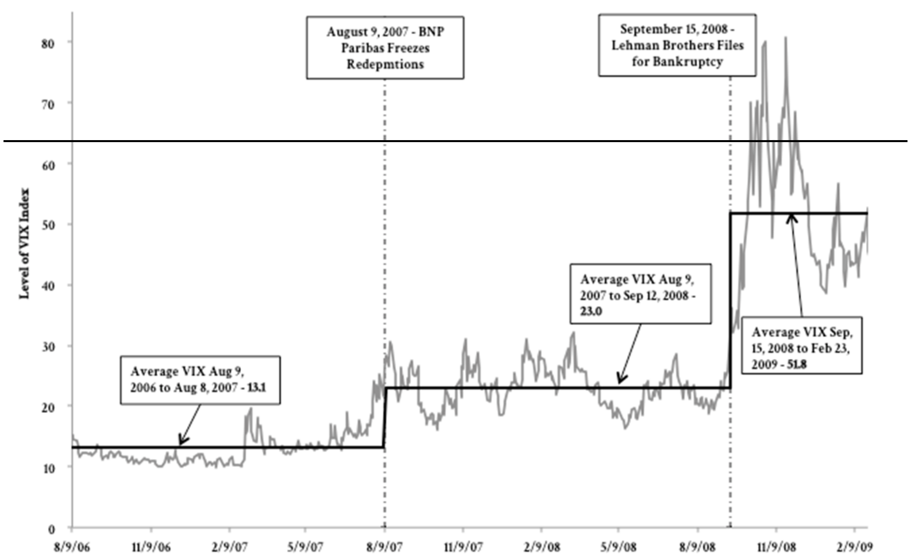
205. *VIX® Index & Volatility*, CHI. BD. OPTIONS EXCH., http://www.cboe.com/micro/vix-and-volatility.aspx (last visited May 5, 2016).

206. *Id.*

expiration dates falling between twenty-three days and thirty-seven days.[207] Figure 1 shows movements in the VIX Index in relation to the dates specified above.

Figure 1

VIX Level over the Financial Crisis Period and One Year Prior
August 9, 2006 to February 23, 2009



The trend is clear: beginning around the time of the BNP decision to freeze redemptions, the VIX Index rose by roughly seventy-five percent, from an annualized level of 13.1 to 23.0 over the period before Lehman's collapse. Following Lehman's failed rescue attempt and ultimate bankruptcy, the VIX Index increased drastically, more than doubling in value over the period from September 15, 2008 to February 23, 2009. The market visibly expected the variance in future returns on the S&P 500 to be an order of magnitude higher than that expected less than a year earlier. With such a dramatic increase in expected volatility, presumably the return series of common stocks constituting the Index would be structurally different from prior period returns. Considering that event studies rely on normally distributed security returns, this precipitous change in variance structure is likely to result in violations of the normality assumption of a properly specified event study model.

207. Chi. Bd. Options Exch., White Paper: The CBOE Volatility Index—VIX 5 (2014), http://www.cboe.com/micro/vix/vixwhite.pdf.

The specification and analysis of power tests in the following Part include data for the twenty-nine securities in the Dow Industrial Index at the time of the financial crisis, less General Motors, which received a government bailout during this period.[208] The return series were downloaded from Bloomberg L.P. using a logarithmic transformation and are adjusted for dividends and stock splits. The VIX Index data were also downloaded from Bloomberg and have been recalibrated as the annualized traded value of the S&P 500 options' implied volatility divided by the square root of 250 (the approximate number of trading days in a year). Thus, the VIX parameter used in the event study analysis is the daily value of the implied volatility. The independent variable in the regression equation is the log return of the S&P 500 Total Return Index, which includes reinvestment of ordinary and special dividends.[209] Summary statistics of the twenty-nine securities used in the analysis are presented in Table 1.

208. FCIC REPORT, *supra* note 197, at 375.

209. S&P DOW JONES INDICES, S&P U.S. INDICES: METHODOLOGY 26 (2016), http://us.spindices.com/indices/equity/sp-500 (to locate, select "Methodology" and follow hyperlink).

Table 1
Summary Statistics of Returns on Dow 30 Companies, 2009

	Name	Ticker	Mean	Standard Deviation	Kurtosis	Skew	Chi-squared	p-value	Normally Distributed?
1	Alcoa	aa	-0.46%	4.58%	7.08	-0.02	34.14	0.00	No
2	AIG	aig	-1.24%	9.18%	35.42	-3.49		0.00	No
3	American Express	axp	-0.42%	4.19%	5.63	-0.27	26.95	0.00	No
4	Boeing	ba	-0.27%	2.70%	6.56	0.38	37.39	0.00	No
5	Citigroup	c	-0.79%	6.93%	11.17	0.23	57.42	0.00	No
6	Caterpillar	cat	-0.29%	2.91%	6.19	0.04	27.96	0.00	No
7	DuPont	dd	-0.23%	2.75%	6.51	-0.36	36.32	0.00	No
8	Walt Disney	dis	-0.17%	2.77%	7.67	0.33	42.93	0.00	No
9	GE	ge	-0.37%	3.27%	6.69	-0.13	32.35	0.00	No
10	Home Depot	hd	-0.17%	2.99%	4.49	0.51	25.16	0.00	No
11	Honeywell International	hon	-0.18%	2.63%	5.05	-0.09	18.98	0.00	No
12	HP	hpq	-0.13%	2.68%	6.65	0.58	45.70	0.00	No
13	IBM	ibm	-0.07%	2.17%	5.53	0.26	25.95	0.00	No
14	Intel	intc	-0.17%	3.02%	4.91	-0.18	19.00	0.00	No
15	Johnson & Johnson	jnj	-0.03%	1.57%	15.08	0.93		0.00	No
16	JPMorgan	jpm	-0.21%	4.97%	7.10	0.05	34.40	0.00	No
17	Coca-Cola	ko	-0.06%	1.96%	11.10	0.87		0.00	No
18	McDonald's	mcd	0.03%	1.93%	5.70	0.11	24.71	0.00	No
19	3M Company	mmm	-0.16%	2.10%	6.36	-0.07	29.41	0.00	No
20	Altria	mo	-0.07%	2.05%	15.53	0.05	69.08	0.00	No
21	Merck	mrk	-0.15%	2.69%	8.91	-0.74	66.74	0.00	No
22	Microsoft	msft	-0.13%	2.82%	8.23	0.35	46.82	0.00	No
23	Pfizer	pfe	-0.13%	2.16%	7.32	-0.22	38.12	0.00	No
24	Procter and Gamble	pg	-0.06%	1.71%	8.88	-0.15	45.60	0.00	No
25	ATT	t	-0.13%	2.54%	8.08	0.73	61.80	0.00	No
26	United Technologies	utx	-0.13%	2.41%	7.44	0.55	49.61	0.00	No
27	Verizon	vz	-0.09%	2.41%	6.93	0.46	42.78	0.00	No
28	Wal-Mart	wmt	0.01%	1.97%	7.46	0.22	38.87	0.00	No
29	Exxon	xom	-0.05%	2.81%	10.61	0.22	54.84	0.00	No

Note: Summary statistics are based on log return series. The test for normally distributed returns is based on the stata command *sktest*. Empty values for the chi-squared test represent "absurdly high numbers" and should be interpreted as highly non-normal.

Table 1 reveals that the security returns for these twenty-nine companies are not normally distributed over the crisis period. Skewness and kurtosis are the third and fourth standardized moments around the mean, and they are "used to describe shape characteristics of a distribution."[210] The normal distribution has a skewness of zero and a kurtosis of three, and the deviations from normality can be described by comparing the values of these moments.[211] To assess whether the returns depart from the normal distribution, I performed the test described by D'Agostino, Belanger, and D'Agostino, Jr. in 1990[212] with the empirical correction developed by Patrick Royston in 1991.[213] The chi-squared results for each security are highly statistically significant, indicating the presence of non-normality. For three securities—AIG, Johnson & Johnson, and Coca-Cola—the return distribution is so non-normal that standard statistical software fails to calculate the statistic.

An alternative means of testing for non-normally distributed data recommended by statisticians is to generate a normal probability plot relating the empirical return series to the normal distribution.[214] Figure 2 generates two normal probability plots, one for a financial firm (AIG) and one for an industrial corporation (Caterpillar).

210. D.N. Joanes & C.A. Gill, *Comparing Measures of Sample Skewness and Kurtosis*, 47 STATISTICIAN 183, 183 (1998).

211. Ralph B. D'Agostino et al., Commentary, *A Suggestion for Using Powerful and Informative Tests of Normality*, 44 AM. STATISTICIAN 316, 317 (1990).

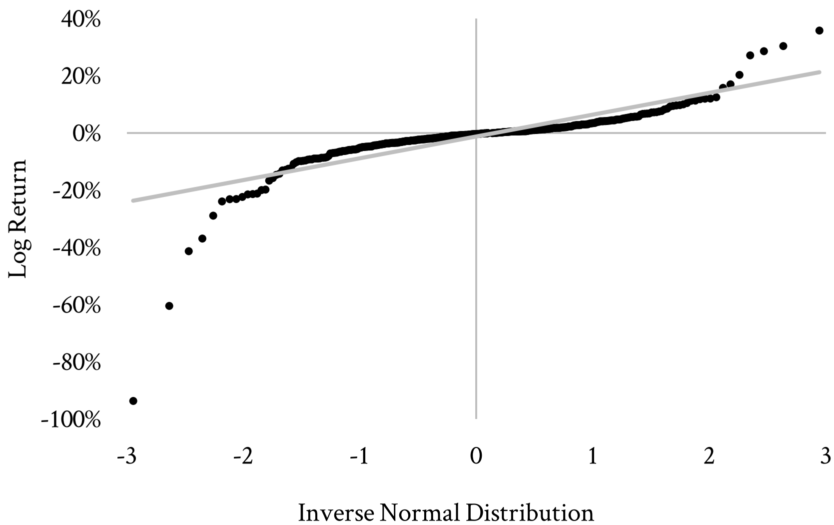
212. *Id.*

213. Patrick Royston, *Comment on sg3.4 and an Improved D'Agostino Test*, STATA TECHNICAL BULL., Sept. 1991, at 23-24.

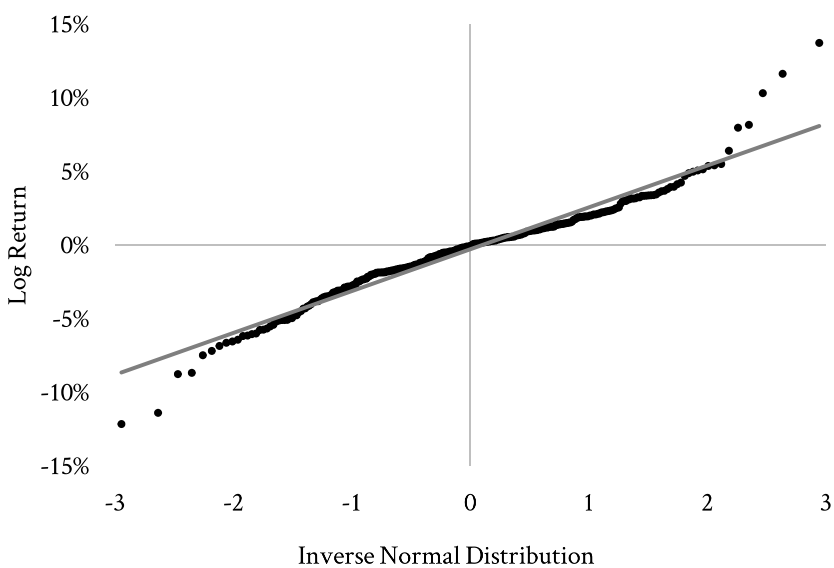
214. *See* D'Agostino et al., *supra* note 211, at 319.

Figure 2

AIG
August 9, 2007 to February 23, 2009



Caterpillar
August 9, 2007 to February 23, 2009



Both charts plot the ordered log security returns over the crisis period against the inverse of the standard normal cumulative distribution. If the return series were normally distributed, the data points would lie along a straight line. Instead, clear evidence exists of the canonical "fat tail" distribution common to financial markets;[215] there are too many very large and very small returns. Given this pattern, there is reason to believe that standard parametric inference tests will cause overrejection of the null hypothesis.

B. Market Models and Event Windows

In conducting the specification and power tests in the spirit of a Brown and Warner study, this Note examines the results of four event study models over the financial crisis: two models previously established in the academic canon—the standard OLS market model and the SQ test proposed by Gelbach—as well as two novel event study models that directly control for changes in market volatility—an OLS market model with a VIX standard error correction[216] and a feasible generalized least squares (FGLS) market model.[217] The VIX-adjusted and FGLS market models adjust for contemporaneous changes in volatility through slightly different econometric methods, and both have been proffered by expert witnesses in federal securities lawsuits. A detailed explanation of the precise model form of each competing methodology can be found in the Appendix to this Note.

Sensitivity to the choice of estimation window is analyzed by comparing results across models with event study estimation windows corresponding to the one-year period preceding the financial crisis (pre-period window),[218] the 250 days directly abutting each calculated abnormal return (rolling window),[219] and the crisis period itself (in-sample window). As noted above, the estimation window is the time period over which the relationship between the return on the security and the return on the market is estimated.

215. Adrian Pagan, *The Econometrics of Financial Markets*, 3 J. EMPIRICAL FIN. 15, 37 (1996).

216. Expert Report of Gregg A. Jarrell at 32-33, *In re* Countrywide Fin. Corp. Sec. Litig., 273 F.R.D. 586 (C.D. Cal. 2009) (No. CV-07-05295-MRP (MANx)), 2010 WL 6681871.

217. Expert Report of Mukesh Bajaj at 45, *In re* Fed. Home Loan Mortg. Corp. (Freddie Mac) Sec. Litig., 281 F.R.D. 174 (S.D.N.Y. 2012) (No. 1:09-MD-2072 (MGC)), 2011 WL 7473682.

218. For all but the SQ test this estimation window is August 9, 2006 to August 8, 2007. The SQ test compares residual returns to the empirically derived critical values and is best conducted using an estimation period with a number of returns that is a multiple of the critical value. *See* Gelbach et al., *supra* note 120, at 523. Using a two-tailed test and a 95% confidence level, we will be looking at the 2.5% and 97.5% critical values. With this consideration in mind, the estimation period is modified to start on August 24, 2006, resulting in a 240-day estimation window and critical level abnormal returns of the 6th and 234th largest returns.

219. Following a similar logic to that discussed in note 218 above, the rolling window consists of 240 trading days for the SQ-test event studies.

Accordingly, the pre-period window regressions estimate the relationship once with data from before the crisis and use the estimated parameters to predict expected returns during the crisis period. Rolling window event studies conduct a separate estimation for each predicted daily return, using only the most recent 250 daily returns. Finally, in-sample event studies use the daily returns from the crisis period itself to estimate predicted returns, with each abnormal return calculated as the residual from the regression procedure.

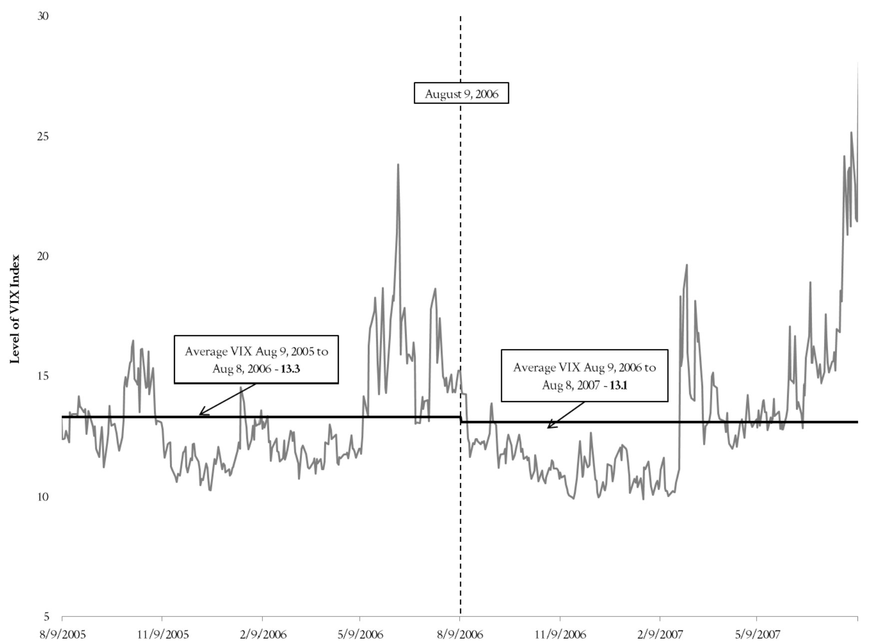
V. Results

Using the models developed in Part IV above, the specification and power tests were applied to the twenty-nine stocks in the sample. For the specification analysis, using the abnormal returns and *t*-statistics from the event study, the average rejection rate was computed over the financial crisis period for each security. A properly specified event study model will have a rejection frequency close to the confidence level determined by the test. Thus, for an event study calculating statistical significance with 95% certainty, approximately 5% of the abnormal returns over the period should be statistically significant. The power test examines the ability of each model to detect abnormal performance when it exists. Here, "artificial" abnormal performance is imputed into the empirical return series, and statistical significance is calculated as to the modified return.

A. Type I Error, Specification Test

For a first-pass analysis, it is helpful to test whether a model performs adequately even in the absence of changes in volatility. A proposed event study technique that modifies the standard to incorporate the effect of marketwide changes in variance should also be able to detect abnormal performance when security returns approach normality. Figure 3 charts the level of the VIX Index over the two years before BNP froze redemption in its investment funds. Although variation exists, there is no general time trend that would be expected to bias the results. Separated into one-year periods, the levels of the VIX Index are nearly equivalent, thus giving no reason to believe that pre-period estimation window results would be biased.

Figure 3
VIX Level over the Two Years Prior to the Start of the Financial Crisis
August 9, 2005 to August 8, 2007



Given this pattern in market volatility over the prior sampling period and the conclusions of Brown and Warner regarding the immateriality of model choice, we would expect to find similar performance across event study techniques. Table 2 demonstrates the specification properties of the four event study models, estimated over the three sampling windows for the one-year trading period preceding the financial crisis.

Table 2
Rejection Frequencies: August 9, 2006 to August 8, 2007
Using a 95% Confidence Interval and Two-Tailed Test

	Pre-Period Window				Rolling Window				In-Sample Window			
	OLS	SQ	VIX Adj.	FGLS	OLS	SQ	VIX Adj.	FGLS	OLS	SQ	VIX Adj.	FGLS
3M	2.0%	3.2%	1.6%	2.0%	2.0%	3.6%	1.6%	2.0%	3.6%	4.8%	4.0%	4.8%
AIG	3.2%	5.6%	4.0%	4.8%	3.6%	4.8%	4.0%	4.8%	4.0%	4.8%	4.8%	4.0%
Alcoa	6.8%	7.2%	6.8%	10.0%	6.4%	7.2%	6.4%	7.6%	6.4%	4.8%	5.2%	7.2%
Altria	2.4%	5.2%	3.6%	4.0%	3.6%	5.2%	3.2%	4.0%	3.6%	4.8%	4.0%	4.8%
American Express	5.2%	4.4%	5.2%	6.4%	6.4%	6.0%	6.4%	8.8%	4.8%	4.8%	4.8%	5.2%
ATT	8.4%	5.2%	9.6%	10.8%	6.0%	4.4%	8.4%	8.8%	5.6%	4.8%	6.4%	10.0%
Boeing	4.0%	4.0%	3.6%	3.6%	4.0%	4.4%	3.6%	4.0%	4.4%	4.8%	6.0%	8.8%
Caterpillar	5.2%	5.6%	4.8%	6.0%	4.4%	5.2%	4.0%	5.6%	3.6%	4.8%	2.8%	6.8%
Citigroup	8.4%	8.8%	7.2%	8.4%	6.8%	6.0%	5.6%	7.2%	6.0%	4.8%	4.8%	6.4%
Coca-Cola	5.2%	4.8%	4.8%	5.6%	5.2%	6.0%	4.4%	5.2%	4.8%	4.8%	2.8%	4.8%
Disney	1.6%	3.6%	3.6%	3.6%	3.6%	3.2%	2.8%	4.0%	5.6%	4.8%	6.0%	8.0%
DuPont	6.0%	5.6%	7.6%	7.6%	6.0%	4.8%	6.8%	9.2%	4.4%	4.8%	5.2%	7.2%
Exxon	6.0%	4.8%	8.4%	9.6%	5.6%	5.2%	8.8%	9.2%	4.8%	4.8%	8.4%	12.7%
GE	5.2%	4.4%	4.8%	6.0%	6.0%	5.2%	5.2%	7.2%	3.6%	4.8%	4.4%	6.8%
Hewlett Packard	0.8%	0.8%	1.6%	2.0%	2.8%	2.4%	2.8%	3.6%	5.2%	4.8%	6.4%	9.2%
Home Depot	5.2%	5.6%	6.0%	7.6%	5.6%	5.2%	5.2%	6.4%	4.8%	4.8%	6.0%	6.8%
Honeywell	3.2%	3.2%	4.0%	4.4%	5.6%	5.2%	4.8%	5.6%	6.4%	4.8%	5.6%	6.0%
IBM	7.2%	4.8%	6.8%	9.6%	8.4%	5.6%	6.8%	9.2%	4.8%	4.8%	4.4%	5.6%
Intel	2.8%	2.8%	3.6%	4.0%	4.8%	4.8%	4.0%	4.4%	5.2%	4.8%	4.4%	7.6%
Johnson & Johnson	3.6%	3.2%	2.8%	3.6%	5.2%	4.8%	5.2%	6.0%	4.8%	4.8%	6.4%	6.4%
JPMorgan	4.0%	4.8%	4.8%	4.8%	4.0%	4.4%	4.4%	4.8%	4.8%	4.8%	5.2%	6.0%
McDonald's	0.0%	0.8%	0.4%	0.4%	2.4%	1.2%	2.4%	3.6%	6.8%	4.8%	7.2%	10.0%
Merck	3.2%	4.0%	3.2%	3.6%	3.6%	4.8%	4.4%	5.2%	3.6%	4.8%	3.6%	5.2%
Microsoft	2.0%	6.0%	2.4%	3.6%	2.0%	4.4%	2.0%	3.2%	6.0%	4.8%	8.0%	10.0%
Pfizer	0.8%	1.6%	1.2%	2.0%	1.6%	2.0%	2.0%	3.2%	2.0%	4.8%	3.2%	6.0%
Procter & Gamble	2.4%	2.8%	2.0%	2.0%	4.0%	4.8%	2.0%	2.8%	5.2%	4.8%	4.0%	3.6%
United Technologies	2.4%	2.4%	3.2%	3.2%	4.4%	5.2%	3.2%	4.0%	5.2%	4.8%	5.2%	8.0%
Verizon	8.8%	8.0%	9.2%	10.4%	7.2%	8.0%	9.2%	10.0%	6.0%	4.8%	6.0%	7.2%
Wal-Mart	5.2%	4.4%	6.0%	7.2%	5.2%	4.0%	5.6%	6.8%	5.2%	4.8%	6.0%	9.2%
Mean	4.2%	4.4%	4.6%	5.4%	4.7%	4.7%	4.6%	5.7%	4.8%	4.8%	5.2%	7.0%
Standard Deviation	2.4%	1.9%	2.4%	2.9%	1.7%	1.4%	2.1%	2.3%	1.1%	0.0%	1.4%	2.1%

The results in Table 2 support the notion that model choice has little effect on the specification properties of an event study when market volatility is stable. Regardless of statistical method or sampling window, the rejection frequencies are close to the expected value given a 95% confidence level test. Although the average rejection rates are comparable, using a rolling window estimation period does reduce the variance in the rejection frequencies across securities. Given that event studies in securities fraud cases are used for a single security, a smaller variance in rejection frequencies would be preferable, all else equal. This would suggest a convergence on the part of individual securities to the desired Type I error level attached to the test.

Table 3 reports the results of the same analysis performed over the financial crisis. Considering the known statistical violations of normality in the return series, rejection frequencies would be expected to diverge significantly from the 5% level stipulated by the test. If the model variants proposed by Jarrell and Bajaj[220] are capable of controlling for the increase in variance, the VIX-adjusted and FGLS event study models should result in rejection frequencies closer to the value determined by the confidence level of the test.

220. *See supra* notes 216-17 and accompanying text.

Table 3
Rejection Frequencies During the Financial Crisis
Using a 95% Confidence Interval and Two-Tailed Test

	Pre-Period Window				Rolling Window				In-Sample Window			
	OLS	SQ	VIX Adj.	FGLS	OLS	SQ	VIX Adj.	FGLS	OLS	SQ	VIX Adj.	FGLS
3M	14.7%	21.1%	0.8%	2.3%	8.2%	9.5%	2.6%	3.9%	5.4%	4.9%	4.6%	6.2%
AIG	64.2%	66.0%	38.7%	36.6%	16.8%	18.8%	12.4%	13.4%	5.2%	4.9%	5.7%	8.2%
Alcoa	22.4%	18.8%	1.8%	2.3%	12.1%	9.0%	5.9%	8.0%	6.4%	5.2%	5.9%	12.1%
Altria	16.5%	20.6%	1.5%	2.6%	12.9%	11.1%	5.9%	7.2%	5.9%	4.6%	6.7%	12.1%
American Express	35.3%	33.2%	8.8%	9.8%	13.9%	12.4%	5.7%	8.0%	5.9%	4.9%	4.1%	9.8%
ATT	14.4%	14.7%	1.5%	1.8%	7.7%	7.5%	4.1%	5.7%	6.4%	4.6%	3.1%	2.8%
Boeing	21.6%	21.9%	3.1%	4.1%	11.9%	10.3%	5.9%	7.7%	5.4%	4.6%	5.2%	7.5%
Caterpillar	7.0%	11.1%	0.3%	0.5%	10.3%	8.5%	3.9%	4.9%	5.9%	4.6%	5.2%	9.0%
Citigroup	60.8%	57.2%	30.7%	34.3%	20.6%	17.8%	11.3%	12.6%	6.7%	4.6%	6.7%	10.8%
Coca-Cola	29.6%	31.4%	6.2%	8.8%	13.7%	12.6%	6.4%	7.5%	4.4%	4.9%	5.7%	8.2%
Disney	18.0%	16.2%	1.5%	1.5%	11.1%	9.0%	3.1%	5.7%	4.1%	4.6%	2.1%	3.1%
DuPont	14.9%	16.8%	1.8%	2.3%	10.1%	8.5%	4.9%	5.7%	5.4%	4.6%	5.9%	7.2%
Exxon	18.6%	20.4%	0.5%	2.3%	11.3%	10.1%	5.2%	7.0%	6.2%	4.6%	6.2%	13.4%
GE	28.6%	34.0%	5.9%	9.5%	10.3%	13.4%	4.6%	5.9%	5.2%	4.6%	5.2%	10.3%
Hewlett Packard	16.8%	15.5%	2.8%	4.9%	10.3%	8.5%	5.7%	7.2%	4.4%	4.6%	6.4%	15.5%
Home Depot	27.3%	25.3%	3.6%	5.2%	10.1%	10.1%	3.9%	5.2%	5.9%	4.6%	6.2%	9.3%
Honeywell	19.1%	16.0%	1.5%	1.8%	8.5%	8.0%	2.3%	3.9%	7.0%	4.6%	4.6%	4.1%
IBM	13.4%	12.4%	2.1%	2.3%	5.7%	6.4%	3.4%	4.1%	6.7%	4.9%	3.9%	5.2%
Intel	14.9%	14.4%	1.5%	1.8%	7.2%	7.5%	2.8%	4.6%	5.4%	4.6%	6.2%	13.4%
Johnson & Johnson	14.2%	12.1%	1.3%	1.8%	9.5%	7.5%	4.4%	4.9%	3.6%	4.6%	4.6%	12.4%
JPMorgan	51.0%	50.5%	21.9%	24.7%	14.7%	14.9%	7.2%	6.7%	4.6%	4.6%	6.4%	12.4%
McDonald's	19.3%	16.2%	2.6%	3.1%	7.7%	8.2%	3.4%	4.9%	4.6%	4.9%	3.9%	8.2%
Merck	13.9%	18.8%	1.3%	2.1%	6.4%	9.3%	2.6%	4.4%	7.2%	4.9%	5.4%	8.2%
Microsoft	24.0%	24.2%	4.9%	7.7%	10.1%	9.8%	5.2%	7.7%	6.2%	4.6%	5.4%	9.5%
Pfizer	8.0%	14.9%	0.8%	1.0%	10.3%	11.6%	5.2%	6.7%	5.2%	4.6%	4.6%	5.2%
Procter & Gamble	18.8%	17.5%	1.8%	2.1%	11.9%	9.3%	4.9%	6.4%	6.2%	4.6%	5.9%	10.6%
United Technologies	17.3%	18.3%	1.5%	1.5%	11.3%	11.3%	4.1%	5.9%	5.4%	4.9%	4.6%	6.2%
Verizon	14.9%	11.9%	1.8%	2.1%	9.0%	8.2%	4.1%	4.9%	5.2%	4.9%	5.7%	8.2%
Wal-Mart	21.4%	18.3%	2.3%	4.1%	9.0%	9.8%	4.9%	7.2%	6.4%	5.2%	5.9%	12.1%
Mean	22.8%	23.1%	5.3%	6.4%	10.8%	10.3%	5.0%	6.5%	5.5%	4.7%	5.1%	8.8%
Standard Deviation	13.9%	13.6%	9.1%	9.3%	3.1%	2.9%	2.3%	2.2%	1.0%	0.1%	1.3%	3.3%

The results in Table 3 offer several immediate insights. First, when conducting event studies over periods with significant time-varying changes in market volatility, using an estimation window with data separated in time from the event date will lead to biased results. Averaging over the sample securities, the standard OLS- and SQ-test-based market models reject the null hypothesis of no abnormal performance more than four times as often as they should, considering the confidence level attached to the test. Even for the alternative models where average rejection frequencies are functionally equivalent to the test standard, variance in the rejection frequencies is too large for reliable inference. For instance, using the VIX-adjusted OLS market model with a pre-period estimation window, the average rejection frequency is 5.3%. However, within that sample there are three securities with a null rejection rate over 20% and four with less than 1%.

Even using the better-specified rolling window estimation period, standard OLS- and SQ-test-based models reject the null hypothesis two times as often as they should. This is ostensibly surprising; according to Gelbach, "[t]he SQ test's asymptotic Type I error rate always equals the analyst's desired significance level."[221] There are several likely reasons for my inconsistent empirical findings. First, instead of using contiguously dated estimation periods in testing their model, Gelbach et al. use a Monte Carlo simulation to randomly select one hundred observations from a seven-year period.[222] If there is a *time-varying* component of market volatility, it will likely be eliminated by randomly selecting dates from a multiyear window. Additionally, their dataset consists of the returns on all securities listed in the CRSP database from 2000 to 2007.[223] In light of the results in Table 2, model choice may not have an effect on inference properties when analyzed over the interlude between two financial market crashes. Although the SQ test may be a reliable statistical method in narrowly described conditions, event studies conducted for securities litigation do not use random samplings or select returns over a seven-year period to generate the estimation period. As a result, even if the SQ test creates asymptotically defined error rates in the abstract, it does not appear to provide a practical advantage over the standard OLS market model when adapted to the necessities of securities fraud class actions.

Using in-sample estimation resolves the issue of overrejection for all risk-adjustment procedures except for FGLS, which has elevated levels of statistically significant results with higher variance.[224] When using a sampling

221. Gelbach et al., *supra* note 120, at 533.

222. *Id.* at 521.

223. *Id.* at 505.

224. Although this could also be an artifact of events in the data, some have found a tendency to falsely reject the null hypothesis when using FGLS in other contexts. *See, e.g.*, Aman Ullah & Xiao Huang, *Finite Sample Properties of FGLS Estimator for Random-Effects Model Under Non-Normality*, *in* PANEL DATA ECONOMETRICS: THEORETICAL CONTRIBUTIONS AND EMPIRICAL APPLICATIONS 67, 83 (Badi H. Baltagi ed., 2006) (finding

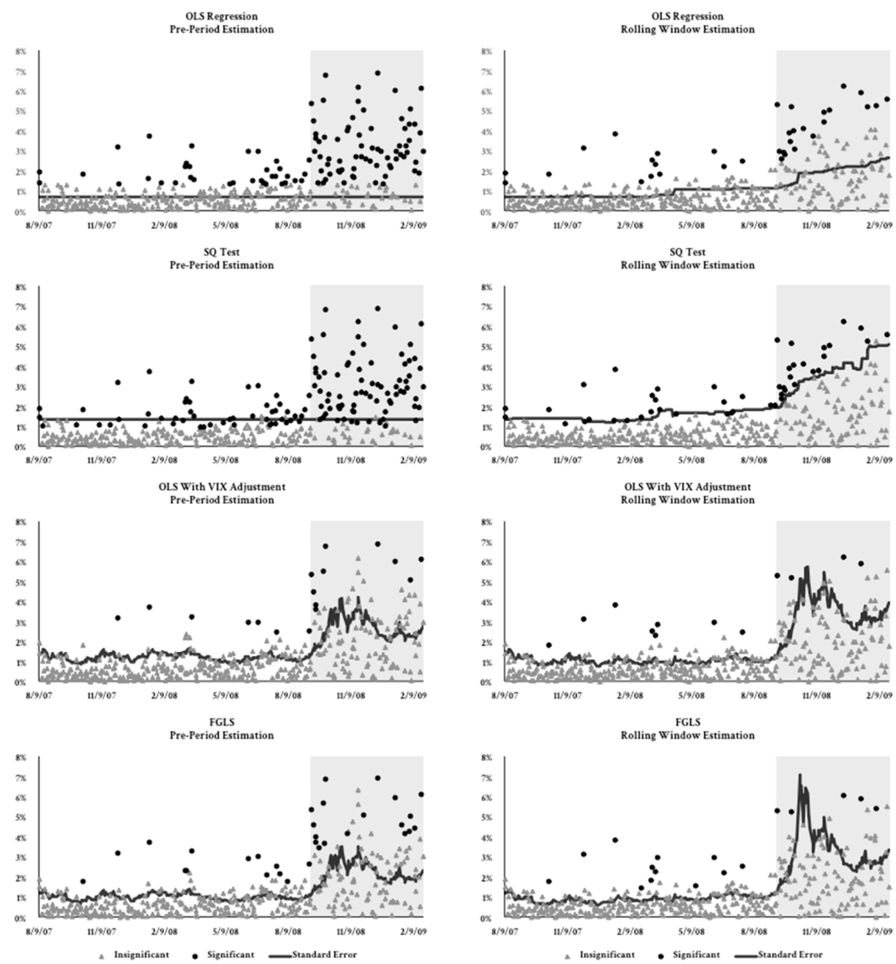
footnote continued on next page

window encompassing the entire event period, any deviation from the desired error rate will be the result of idiosyncratic deviations in the shape of the return distribution, not a result of changes in market volatility. However, given the exigencies of litigation, courts are unlikely to allow expert testimony to use postevent return data to explain the variation in security returns surrounding dates of misrepresentation or corrective disclosure. Table 3 suggests this is not necessary; it is possible to obtain statistically robust results by using a volatility-corrected event study and a rolling window estimation period. Graphical depiction allows for a more complete understanding of the ability of the models to isolate abnormal performance and statistical significance. Figure 4 displays the abnormal returns and variance estimates for General Electric[225] over the financial crisis period.

the use of FGLS in the presence of non-normal time-varying errors in a random-effects panel-data model to cause overrejection of the null hypothesis).

225. While General Motors is an admittedly arbitrary choice, the general pattern exhibited in Figure 4 is similar regardless of security chosen. Charts for other securities are available upon request.

Figure 4
Abnormal Returns over the Financial Crisis Period by Model and Estimation Window



The left and right panels of Figure 4 present the abnormal return series for pre-period and rolling window event studies, respectively. Light triangles represent statistically insignificant abnormal returns, dark circles represent statistically significant abnormal returns, the bolded line indicates the daily root mean standard error used to calculate the *t*-statistic, and the gray shaded area indicates the portion of the event period following the collapse of Lehman

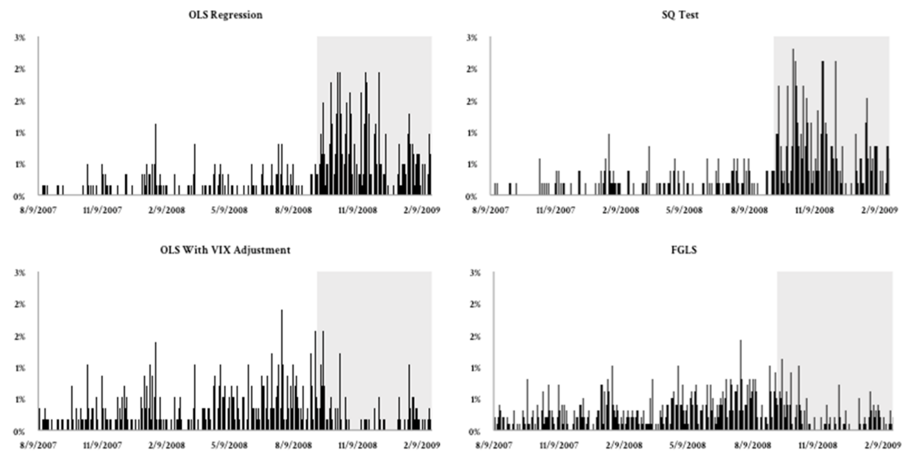
Brothers.[226] As previously demonstrated through tabular results, pre-period estimation and standard models result in excess null rejections. Figure 4 also demonstrates that not only are there too many statistically significant abnormal returns, but they are also clustered in the latter, more volatile portion of the crisis period.

Additionally, Figure 4 demonstrates that this discrepancy in the number of statistically significant rejections in the two phases of the crisis period exists with rolling window uncorrected event studies as well. The improved performance in models incorporating a market proxy is largely a result of the sensitivity of the standard error estimate used to calculate significance. The bolded lines representing the variance of the abnormal returns for the VIX-adjusted and FGLS models increase more rapidly, and to a greater extent, than do those in the standard OLS market model or the SQ test. Accordingly, when return variance increased drastically following the collapse of Lehman Brothers, the abnormal return calculations failed to reflect the total effect of extrinsic volatility. The standard error estimates for the unadjusted models increased slowly and took longer to revert once the market panic subsided. This latter point is significant because it indicates that a properly adjusted event study has the potential to benefit plaintiffs or defendants, depending on where the relevant abnormal return is situated in relation to changes in market volatility. For returns occurring *after* a period of excess volatility, the unadjusted event studies will have a higher standard error and less sensitive *t*-statistics than the corrected models.

This raises an interesting issue often overlooked in the comparative methodology literature: in addition to finding rejection frequencies approaching asymptotic values in the aggregate, we are also concerned with isolating the *right* statistically significant abnormal returns. Not only do the VIX-adjusted and FGLS market models have lower rejection frequencies, but their statistically significant abnormal returns are spread more evenly across the event period, a result expected with normally fluctuating return series. Figure 5 presents another means of evaluating this concern. To compare across models with similar rejection frequencies, the charts in Figure 5 refer solely to the in-sample estimation window models, which have more comparable rejection rates. Aggregating across the twenty-nine securities, each bar represents the percentage of statistically significant returns per day as a percentage of the total number of statistically significant returns during the crisis period. This transformation permits an untainted comparison of the distribution of excess returns over time and across models.

226. The SQ test does not use a standard error to calculate its test statistics. Instead, the bolded line represents the average of the absolute value of the 2.5th- and 97.5th-percentile abnormal returns. The level cannot be directly compared to parametric-based models, but the change in value reflects a common trend.

Figure 5
Relative Occurrence of Abnormal Returns over the Financial Crisis by Model



Standard OLS and SQ test market models result in noticeable clustering of statistical significance in the more volatile portion of the crisis period. Meanwhile, the adjusted event study models in the lower panel of Figure 5 have null rejections distributed consistently across the period. If we were to assume that fraudulent misstatements occur through time with equal probability, then volatility-controlled market models will be more precise in determining statistical significance. Taken together, these results suggest that event study specification during the crisis period is improved with rolling window event studies and models controlling for changes in market volatility.

B. Type II Error, Power Test

I next use an analysis that imputes artificial negative abnormal performance and tests the ability of competing models to detect the imposed excess return to compare the power properties across models. Historically, tests of statistical power have used simulation analysis to distribute the artificial performance randomly across date-security combinations.[227] When used to compare cross-sectional event studies, which include multiple securities, this approach is necessary due to the sheer number of possible combinations.[228]

227. *See, e.g.*, Brown & Warner, *supra* note 4, at 6.

228. For example, in our context there are 29 securities in our sample and 388 trading dates in the financial crisis period. To capture each date-security combination in a cross-sectional event study would require 388^{29} separate event studies for each level of abnormal performance.

However, such a constraint is not present in the context of single-firm, single-event studies—here, each level of abnormal performance has only 388 potential trading dates and twenty-nine securities. Any attempt to simulate the occurrence of the artificial return will only subtract precision from an analysis that tests each date for each firm. Thus, for every security, date, and model combination, the analysis subtracts increasing levels of artificial abnormal return performance and determines the statistical significance of the return using a one-tailed hypothesis test. Table 4 presents the percentage of statistically significant rejections averaged across models.

Table 4

	Rejection Frequencies (1% Abnormal Return)		
	Pre-Period	Rolling Window	In-Sample
Standard OLS	36.6%	24.5%	10.7%
SQ Test	42.9%	29.5%	13.2%
VIX-Adjusted OLS	9.9%	14.1%	15.5%
FGLS	12.5%	18.0%	22.9%
	Rejection Frequencies (2% Abnormal Return)		
	Pre-Period	Rolling Window	In-Sample
Standard OLS	66.7%	54.1%	30.1%
SQ Test	71.7%	59.2%	35.2%
VIX-Adjusted OLS	27.4%	38.3%	39.2%
FGLS	33.9%	45.1%	49.8%
	Rejection Frequencies (3% Abnormal Return)		
	Pre-Period	Rolling Window	In-Sample
Standard OLS	85.2%	75.3%	57.3%
SQ Test	87.5%	78.5%	62.4%
VIX-Adjusted OLS	50.4%	61.1%	60.7%
FGLS	57.2%	67.3%	69.0%
	Rejection Frequencies (5% Abnormal Return)		
	Pre-Period	Rolling Window	In-Sample
Standard OLS	96.6%	92.6%	84.7%
SQ Test	97.1%	93.5%	86.8%
VIX-Adjusted OLS	78.3%	84.4%	82.0%
FGLS	82.1%	87.8%	86.1%
	Rejection Frequencies (10% Abnormal Return)		
	Pre-Period	Rolling Window	In-Sample
Standard OLS	99.6%	98.5%	95.8%
SQ Test	99.6%	98.7%	97.1%
VIX-Adjusted OLS	96.9%	96.7%	95.7%
FGLS	97.8%	97.3%	97.1%

These results reflect the tradeoff between asymptotic size (the Type I error rates found in Table 3) and power (the Type II error rates found in Table 4) that often accompanies tests of inference. As Gelbach et al. note, controlling for the probability that a test incorrectly rejects the null hypothesis can also cause the test to fail to reject the null when it is in fact false.[229] As a result, standard models, which had rejection rates under no abnormal performance considerably higher than those predicted by the significance level of the hypothesis test, are more capable of detecting abnormal performance than the VIX-adjusted or FGLS models. This again is intuitively unsurprising; if standard models overreject the null hypothesis without imputed abnormal performance, they are likely to reject it at a higher rate when abnormal performance is present. Additionally, for all levels of artificial abnormal performance and for each choice of estimation period, the FGLS market model has more statistical power than the VIX-adjusted OLS model. As mentioned in the methodological explanation, this follows from the theory underlying the volatility correction. FGLS event studies more accurately detect abnormal performance because they apply the correction according to the sensitivity of each security to changes in market volatility. The VIX-adjusted OLS model, on the other hand, assumes that changes in VIX affect each security equally, causing under- and overcorrection depending on the stock's sensitivity.

Table 4 further demonstrates that rolling window event studies are more powerful than other volatility-corrected models, but the difference in performance narrows as the size of the imputed abnormal performance increases. This is an encouraging result because the statistical power in tests with a sample of one is generally low.[230] It is not obvious that a model should be expected to detect 1% abnormal performance given the level of variance in individual security returns in modern financial markets. At 3% abnormal return or higher, the discrepancy is manageable. Thus, a court facing a choice between models must weigh competing demands and sacrifices between power and precision, along with the disparate effects each would have on parties to the suit.

C. Robustness Check with S&P 500 Data

There is always a possibility that the increase in rejection frequency over this period is a result of firm-specific events and not exogenous market effects. If this were true, the assumption that rejection rates should converge to the size of the test is invalid. By not accounting for genuine events in the data series, the abnormal return distribution may be more or less normal than if the

229. Gelbach et al., *supra* note 120, at 498.

230. Sanjai Bhagat & Roberta Romano, *Event Studies and the Law: Part I; Technique and Corporate Litigation*, 4 AM. L. & ECON. REV. 141, 149 (2002).

abnormal returns of actual events were accounted for.[231] To ensure that these analyses were not isolating spurious results, I compared the results to those derived from a larger set of securities.

With this concern in mind, the returns of the underlying securities in the S&P 500 were evaluated over the financial crisis period. With a broad cross-section of securities across industry groups, it is reasonable to assume that the true error rate should be the size of the confidence test because, for any given security, it is just as likely that there will be fewer "events" in a certain period than more. When the size of the sample increases with additional securities, the potential departure from the asymptotic error rate is diversified away.

In constructing the sample set, the S&P 500 constituents were first downloaded from the Compustat database, a division of S&P Capital IQ. All historical constituents that entered the Index after August 9, 2007 or were removed from the Index before February 23, 2009 were excluded. The resulting set consisted of all securities that were a part of the Index over the entirety of the crisis period. Using the ticker symbols in the Compustat dataset, individual security returns were downloaded from the CRSP Daily Stock File archive. Duplicates from dual-listed shares and all return series lacking complete data for the full year prior to the estimation period were removed, resulting in a total of 418 stocks in the sample. Table 5 reports the average rejection frequency across securities using a two-tailed test and a 95% confidence level.

Table 5
Rejection Frequencies—Financial Crisis Period

	Pre-Period	Rolling Window	In-Sample
Standard OLS	22.5%	10.3%	5.3%
SQ Test	22.6%	9.9%	4.6%
VIX-Adjusted OLS	4.4%	4.7%	5.1%
FGLS	5.1%	6.0%	9.1%

The findings in Table 5 are nearly identical to the aggregated results presented in Table 3 for the Dow 30 companies. This evidence strengthens the conclusion that the overrejection of the null hypothesis prevalent during the financial crisis is a result of exogenous market factors common to all securities and is not the result of firm-specific events present in a smaller sample of Dow Index stocks.

231. *See* Ford & Kline, *supra* note 180.

Conclusion

Heightened judicial review of expert evidence in securities fraud cases will not require a new procedural framework—flawed event studies already violate the established standard for admissibility outlined in *Daubert v. Merrell Dow Pharmaceuticals, Inc.*[232] In *Daubert*, the Supreme Court relaxed the *Frye* "general acceptance" standard that governed for most of the twentieth century and required academic consensus for expert testimony provided to courts.[233] In exchange for broadening the scope of admissibility, the Court reinforced the historical "gatekeeping role" of the judiciary in filtering evidence provided to the jury[234] and handed down a set of considerations for judicial review—namely that the evidence provided relate to the issue in the case, that the expert be qualified to testify on the subject at hand, and that the proposed testimony be "supported by appropriate validation."[235]

Under the last prong, trial courts are now instructed to "examine the methodologies and principles underlying proffered expert testimony to determine whether those principles and methods are sufficiently valid to admit."[236] The *Daubert* Court provided that in evaluating the basis for testimony, judges might consider "whether it can be (and has been) tested,"[237] whether it "has been subjected to peer review and publication,"[238] the "*known or potential rate of error*" of the scientific technique,[239] and its degree of "general acceptance."[240] A recent study found that judicial error-rate analysis is common and is strongly predictive of admissibility decisions,[241] suggesting that courts may find such an attack in this context persuasive. Although courts are led to believe that a confidence level attached to an event study test corresponds to a known error rate, empirical evidence of deviations from normality indicates that there is often an actionable claim to the contrary.

232. 509 U.S. 579 (1993).

233. *See* Lawrence B. Ebert, Frye *After* Daubert*: The Role of Scientists in Admissibility Issues as Seen Through Analysis of the DNA Profiling Cases*, 1 U. CHI. L. SCH. ROUNDTABLE 219, 219, 224 (1993).

234. *Daubert*, 509 U.S. at 597.

235. David L. Faigman, *The* Daubert *Revolution and the Birth of Modernity: Managing Scientific Evidence in the Age of Science*, 46 U.C. DAVIS L. REV. 893, 902-03 (2013) (quoting *Daubert*, 509 U.S. at 590).

236. *Id.* at 903.

237. *Daubert*, 509 U.S. at 593.

238. *Id.*

239. *Id.* at 594 (emphasis added).

240. *Id.*

241. John B. Meixner & Shari Seidman Diamond, *The Hidden* Daubert *Factor: How Judges Use Error Rates in Assessing Scientific Evidence*, 2014 WIS. L. REV. 1063, 1067.

Moreover, judicial reliance on event study analysis is liable to increase in the future. In *Halliburton II*, the Supreme Court declared that defendants are afforded the opportunity to rebut the presumption of reliance prior to class certification by demonstrating a lack of price impact associated with the alleged misrepresentations or omissions.[242] Given that the overwhelming majority of securities fraud suits settle following certification, bringing the issues of price impact and materiality to the certification stage significantly increases the need for statistically reliable results. Although courts have traditionally been inclined to accept the probative value of expert-provided event studies—perhaps due to deference to technical expertise—a reexamination of the statistical foundations of the test is in order. With the merits hinging on the provision of an event study to satisfy the legal predicates for a cause of action, the legal community should ensure that its faith in econometric objectivity is not based on false pretense.

This Note establishes that event study models failing to explicitly correct for the increased variance in security returns will be biased towards finding statistically significant abnormal returns in the presence of a shift in market volatility. This is not merely an interesting exercise in econometric nuance; 206 class actions were tied to the credit crisis, with billions of dollars in potential corporate liability.[243] As a formal condition for a sustained cause of action, an event study was likely proffered in every case that withstood a motion to dismiss.

Although the effect of event study model choice is less pronounced during periods of stable returns, it is precisely during episodes of market volatility that the findings of an event study are most consequential. Courts should be unwilling to cede their adjudicative role in these disputes to the results of an empirical analysis with unknown or unascertained rates of error. While event

242. 134 S. Ct. 2398, 2414 (2014) ("Even if plaintiffs need not directly prove price impact to invoke the *Basic* presumption, Halliburton contends that defendants should at least be allowed to defeat the presumption at the class certification stage through evidence that the misrepresentation did not in fact affect the stock price. We agree.").

243. *See* CORNERSTONE RESEARCH, SECURITIES CLASS ACTION FILINGS: 2014 YEAR IN REVIEW 4 fig.2 (2014). Liability in these suits is hard to measure because nearly all settle or get dismissed. The Cornerstone report uses two measures that proxy for corporate exposure: the maximum dollar loss (MDL) and the disclosure dollar loss (DDL). The MDL measures the dollar-value change in market capitalization of the security as the difference in the maximum value over the class period and the value on the trading date immediately following the end of the class period. *Id.* at 7. The DDL measures the difference in the market capitalization of the security between the trading day immediately prior to the end of the class period and the trading day immediately afterwards. *Id.* at 6. According to Cornerstone's research, the aggregate MDL for credit-crisis-related filings was over $1 trillion, and the more restrictive DDLs were $174 billion. *Id.* at 6-7 figs.4 & 5. The likely aggregate exposure value is somewhere between those two figures.

studies commonly submitted in federal court likely produce biased results, readily available modeling changes can provide more robust statistical inference. Although these models add a level of methodological complexity to the standard approach, courts would be well advised to consider such modification given the considerable financial exposure generated through private securities fraud suits.

Appendix

The following Appendix describes the precise specification used for the four risk-adjustment methodologies used for the substance of this Note. The Stata code used to produce the empirical results is available at http://works.bepress.com/andrew_baker.

A. Standard OLS Market Model

The standard event study model was broadly outlined in Section III above, but below is the methodology as specifically applied to this analysis.

1. Estimate the market model equation:

$$R_{i,t} = \alpha_i + \beta_i (SP500)_t + \varepsilon_{i,t} \qquad (1)$$

where R is the return on the security, i is a unique firm identifier, and t represents the time component.

2. Derive the daily abnormal return:

$$AR_{i,t} = R_{i,t} - \hat{\alpha}_i - \hat{\beta}_i (SP500)_t = \partial_{i,t} \qquad (2)$$

3. Calculate the *t*-statistic for each abnormal return:

$$tstat_{i,t} = \frac{AR_{i,t}}{SD_i} \qquad (3)$$

where SD_i is the root mean squared error from the equation in (1):

$$SD_i = \sqrt{\frac{\sum_1^T \partial_{i,t}{}^2}{T-1}} \qquad (4)$$

T is the number of observations in the estimated model.

4. Test the abnormal return for statistical significance:

An abnormal return will be statistically significant at the 95% confidence level if:

a) *Two-tailed test:*

$$|tstat_{i,t}| \geq tdist(T, \frac{\alpha}{2})$$

b) *One-tailed test:*

$$Positive\ Test: tstat_{i,t} \geq \text{tdist}(T,\alpha)$$
$$Negative\ Test: tstat_{i,t} \leq tdist(T,\text{-}\alpha)$$

For a two-tailed test and a 95% confidence interval, the null hypothesis of no abnormal return can be rejected if the absolute value of the *t*-statistic is greater than or equal to roughly 1.96. For one-tailed test the critical value is approximately ±1.645 depending on whether you are checking for a positive or negative expected abnormal return.

B. SQ Test

The SQ Test follows the same initial two steps as the standard OLS model, but rather than determining statistical significance through the parametric properties of the t-distribution, it compares the event period abnormal return to the empirical distribution of pre-event fitted excess returns.[1] Following steps one and two above;

1. Sort the abnormal returns from the estimation period from greatest to least:

$$\overline{\overline{AR}}_{(1)} \geq \overline{\overline{AR}}_{(2)} \geq \ldots > \overline{\overline{AR}}_{(T)}$$

2. Determine the statistical significance of the event abnormal return in reference to the sample quantiles implied in (3):

An abnormal return will be statistically significant if:

a. *Two-tailed Test:*

$$AR_{i,t} \geq \overline{\overline{AR}}_{T*\left(\frac{\alpha}{2}\right)} \text{ or } AR_{i,t} \leq \overline{\overline{AR}}_{T*\left(1-\frac{\alpha}{2}\right)}$$

b. *One-tailed Test:*

$$Positive\ Test: AR_{i,t} \geq \overline{\overline{AR}}_{T*(\alpha)}$$
$$Negative\ Test: AR_{i,t} \leq \overline{\overline{AR}}_{T*(1-\alpha)}$$

244. Gelbach et al., *supra* note 120, at 495.

For a two-tailed test and a 95% confidence level with 240 pre-fitted excess returns, the null hypothesis will be rejected if the abnormal return is either greater than or equal to the sixth largest pre-fitted excess return (240*0.025) or less than or equal to the 234^{th} largest pre-fitted excess return (240*0.975). A one-tailed test will reject the null hypothesis if the abnormal return is greater (less) than or equal to the 12^{th} (228^{th}) largest pre-fitted excess return.

C. OLS Market Model with VIX Standard Error Adjustment

The VIX-adjusted OLS market model is in most respects identical to the standard OLS market model with an additional adjustment to the denominator of the *t*-statistic to reflect the difference in the VIX level between the estimation period and the event period. This is a modified version of the model used by Professor Gregg Jarrell as the expert witness for plaintiffs in *In re Countrywide Financial Corporation Securities Litigation.*[2] Following steps (1) and (2) above, along with the root mean squared error from (4):

1. Calculate the daily VIX adjustment factor:

$$Vadj_t = \frac{VIX_t}{\left(\frac{\sum_1^T VIX_t}{T}\right)} \tag{5}$$

which is the ratio of the event date VIX to the average VIX over the estimation window.

2. Calculate the modified *t*-statistic as:

$$tstat_{i,t} = \frac{AR_{i,t}}{SD_i * Vadj_t} \tag{6}$$

3. Test the abnormal return for statistical significance following Step 4 of the standard OLS market model.

D. FGLS Market Model

The FGLS market model follows the underlying intuition of the VIX-adjusted OLS model—if a change in market volatility results in violations of the normality assumption, ignoring it while modeling expected returns will

245. Expert Report of Gregg Jarrell at 32-33, 2010 WL 6681871 (C.D. Cal.) (Mar. 31, 2010). Jarrell used the implied volatility of a set of firms within a constructed peer index rather than the VIX Index to calculate the ratio adjustment. He also uses ratio adjustments based on average volatility levels of sub-periods of the Class Period, whereas this paper calculates a separate adjustment factor for each analyzed trading day.

result in biased inference. FGLS is an attractive alternative to OLS when there is evidence of heteroscedasticity,[3] which causes the standard errors of OLS to become inflated. As a method to model the function of heteroscedasticity with empirical data, FGLS is consistent and asymptotically more efficient than OLS.[4] Using FGLS to adjust an event study for volatility is notionally superior to the ad hoc modification used in the VIX-adjusted OLS market model described in Part IV(B)(3) above. Rather than assuming a constant effect across securities, FGLS allows the adjustment factor to vary across firms depending on the historical relationship between the abnormal returns and the market volatility proxy. However, FGLS is a slightly more complicated statistical procedure, and as such may be more difficult to explain to a judge and jury. The FGLS model has been used by Mukesh Bajaj as an expert witness for the defendants in *In re Federal Home Loan Mortgage Corp. (Freddie Mac) Securities Litigation*.[5] The steps in conducting an FGLS market model are as follows:

1. Estimate the market model equation:

$$R_{i,t} = \alpha_i + \beta_i(SP500)_t + \varepsilon_{i,t} \tag{7}$$

2. Using the residuals from the market model equation, estimate the variance equation:

$$\partial_{i,t}{}^2 = \beta_i(VIX_t)^2 + \eta_{i,t} \tag{8}$$

3. Recalculate the market model equation in (7) using weighted least squares, with the weight used being the inverse of the predicted value from the variance equation.

$$\begin{aligned} R_{i,t} &= \alpha_i + \beta_i(SP500)_t + \vartheta_{i,t} \\ weight_{i,t} &= \frac{1}{(\widehat{\beta_i} * VIX_t{}^2)} \end{aligned} \tag{9}$$

246. Heteroscedasticity represents a violation of the assumption that the modelling errors (in the context of an event study, the abnormal return) are uniform and constant across time, which is a predicate for OLS regression. *See* THE SAGE ENCYCLOPEDIA OF SOCIAL RESEARCH METHODS 464 (Michael S. Lewis-Beck eds. 2004).

247. *See* JEFFREY M. WOOLDRIDGE, INTRODUCTORY ECONOMETRICS: A MODERN APPROACH 284 (2006).

248. Expert Report of Mukesh Bajaj at 45, 281 F.R.D. 174 (Aug. 15, 2011). The reader should be aware that I have previously worked with Mukesh Bajaj.

4. Repeat steps 2 and 3 until $\widehat{\alpha_\iota}$ and $\widehat{\beta_\iota}$ converge

5. Calculate the *t*-statistic based on the abnormal return and predicted residuals from (9) and (8) respectively.

$$tstat_{i,t} = \frac{\vartheta_{i,t}}{\sqrt{(\widehat{\ }_\iota * VIX_t{}^2)}}$$

6. Test the abnormal return for statistical significance following Step 4 of the standard OLS market model.